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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1386

Proceedings of the Symposium on Observation, Analysis, and Space Research Applications of the Lunar Motion

Held at the Boeing Scientific Research Laboratories
Seattle, Washington
August 19, 1968

Edited by
J. Derral Mulholland

JET PROPULSION LABORATORY
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Preface

This report was edited and produced under the cognizance of the Systems Division of the Jet Propulsion Laboratory.

Events of recent years have returned the moon to a position of considerable scientific interest. For no aspect of lunar studies has this been more true than for the study of the geocentric motion of the moon. Classical astronomers, spacecraft analysts, and those who work on the interface between these two groups have completely new data types that hold the promise of order-of-magnitude improvements in the knowledge of the various aspects of the lunar motion, as well as of physical properties of the moon. This symposium gave another opportunity for personal contact between those individuals representing the full diversity of current working interest in the lunar motion, in the expectation that this co-mingling would lay the groundwork for additional progress.

I thank Dr. Harold Liemohn for his efforts as co-coordinator of the symposium, particularly his splendid handling of the physical arrangements, Dr. Zdenek Kopal for exercising his knowledge and charm as our chairman, Dr. John Noyes for his cooperation in making the Laboratory, of which he is Director, available for this purpose, and the staff of the Boeing Scientific Research Laboratories for helping to make the meeting a success. Special appreciation is due Mrs. Carol Hilbert for typing the manuscript with her usual competence and good humor.

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Introductory Remarks

Zdenek Kopal
Symposium Chairman
University of Manchester

Ladies and gentlemen, friends of the moon, welcome to Seattle. We will hear a great deal about the moon today, and this is cause for a great deal of rejoicing on the part of many of us here who would qualify as veterans of the subject. It is indeed a fact that celestial mechanics, in general, and the motion of the moon, in particular, has undergone a wonderful renascence in the past few years. This is a subject which, toward the end of the life of E. W. Brown 20 or 30 years ago, seemed to be almost played out. Recently, it has blossomed out in so many new directions and posed so many new problems that it certainly has never been more alive in the past than it is at the present time. The reasons are obvious. As always in the history of science, not only of astronomy, these periods of renascence come when advancing technology places in the hands of the scientist new technical means at his disposal. This is applicable to our field at the present time.

In the past, celestial mechanics was largely concerned with the interpretation of the measured coordinates x and y projected on the plane of the sky. Now, these are of very subordinate importance to us, in fact largely unknown in many instances. We have now to work with r and $\dot{r}$ given, not by conventional telescopes, but by range and doppler tracking, which exceeds the accuracy with which astronomers can measure position at the present time. Secondly, the differential equations of this problem which, in their more rigorous form, are formidable, need no longer be handled by expansions. This was a hopeless task that culminated sometime near the turn of the century, when the literal lunar theory was developed in something close to 1,000 periodic terms. Somewhere between 800 and 900 terms became a no-man's land where no investigator would agree with anyone else. It took years to check out a particular term. This is unnecessary now, thanks to the wonderful computers that have come into being during the past 20 years or so.

I recall the story, which takes us back 30 years or more, when the aging E. W. Brown was taken to the Watson Computing Laboratory at Columbia University and shown by the young Eckert the way in which his computations of the terms of the lunar theory could be done by punched card machines. In the beginning, Brown did not really believe it, and his coming down from New Haven to New York was only an act of faith. Then Eckert produced the goods, and E. W. Brown confessed to those present that what he really saw was, by comparison, a joy. What impressed him was the speed of computation, ten multiplications a second, which was something unheard of in the days of his generation.

At present, we have exceeded this speed of computation by approximately 10^5, and this has opened the horizons as to what can be done beyond the wildest dreams of Brown's generation. I am anticipating that a large part of what we will hear today will be concerned with the amplification of this progress. I expect that what we shall hear today will be only in the nature of a progress report, because the field is in such a rapid state of progress that it is very difficult to be over-optimistic.

N69-34634

Lunar Orbiter Photo Site Location

T. J. Hansen
Space Division, The Boeing Co.

Abstract[1]

Accuracy in defining the locations of Lunar Orbiter *photographs is being increased through improved data analysis techniques. Consistency in the location of surface features photographed during several missions is used as the criterion for judging accuracy of the techniques. Orbit determination, attitude maneuvers, lunar radius, and selenographic coordinate system orientation enter into the investigation.*

Discussion

Sjogren: I do not think you mentioned that the lunar radius solution was a dynamical solution, so that it was from a center of gravity rather than a geometrical center.

Hansen: Yes, the radius is from the center of gravity, as defined by the ephemeris.

Sjogren: Were the large corrections in the orbit determination caused by some discrepancy between Ephemeris Time (ET) and Universal Time (UT)?

Hansen: A large part of it was the handling of ET–UT corrections. Using a more consistent procedure from one mission to another also causes an improvement.

Eichhorn: Did you impose a condition on the reductions that the coordinates or features common to different flights must always come out the same? Were the coordinates introduced as unknowns in the reduction procedure, or did they come from a parameter estimation that did not involve the information that they really are the same?

Hansen: We did investigate how a variation in the moon's orientation enters into the problem.

Eichhorn: No, this is not what I mean. Now, you measure the coordinates of one particular feature from various passes. When you have your final results, do you then superimpose the condition that the coordinates of any particular feature have to be the same? That is, did you reduce the various frames to each other, or did you reduce each frame individually and take the mean of the individual results to get your final results?

Hansen: Each frame is reduced individually.

Sconzo: What Dr. Eichhorn says is that you must put a constraint on your solution, because the distances are fixed and must not be varied.

[1]Abstract only (no manuscript available).

Discussion (contd)

Hansen: No, it is merely a measure we have of the accuracy of the results.

Eichhorn: The reason I asked is because this has recently become an accepted practice in astrometry, when positions are obtained from more than one frame, that the frames be reduced individually and also to each other. I was curious to see if this was also applied to the moon mapping here.

Hansen: We considered using that technique, but decided that it would not be possible with the funds available for this purpose. However, I think that technique is used by ACIC.

Lundquist: Would you care to comment or conjecture on the 2-km error that remains? What do you think is the cause of it?

Hansen: We think a significant problem is still in the orbit determination, specifically the inability to simulate the gravitational model.

Van Flandern: Which lunar ephemeris did you use?

Mulholland: They were using LE 4 and LE 5. Incidentally, I would caution against inferring anything about the adequacy of the ephemeris from these results, because this work is fairly insensitive to the ephemeris.

Sjogren: I did not notice any signs on your differences. Were they random?

Hansen: They are not random. There is very definitely a bias in the variation from the mean values.

Sjogren: In-track or latitude biases?

Hansen: We can see no particular pattern. They are just biases.

Kopal: Did you account for the optical aberrations mathematically?

Hansen: The optical aberration of the lens systems is of very minor significance compared with the other inaccuracies that we had to work with. We did not account for it.

Kopal: To what extent can your accuracies be influenced by finite exposure times, that is, the accuracy with which you can assign a position in the orbit that corresponds to the photograph?

Hansen: We re-determined the photograph times as a check on the initial determinations, and we feel that they are accurate to within 0.1 s.

Kopal: That is 200 meters. Would that not result in the same uncertainty in the position from which the photographs were taken? What influence will that have on your results?

Hansen: We did not investigate it because we thought it was a small part of the total discrepancy.

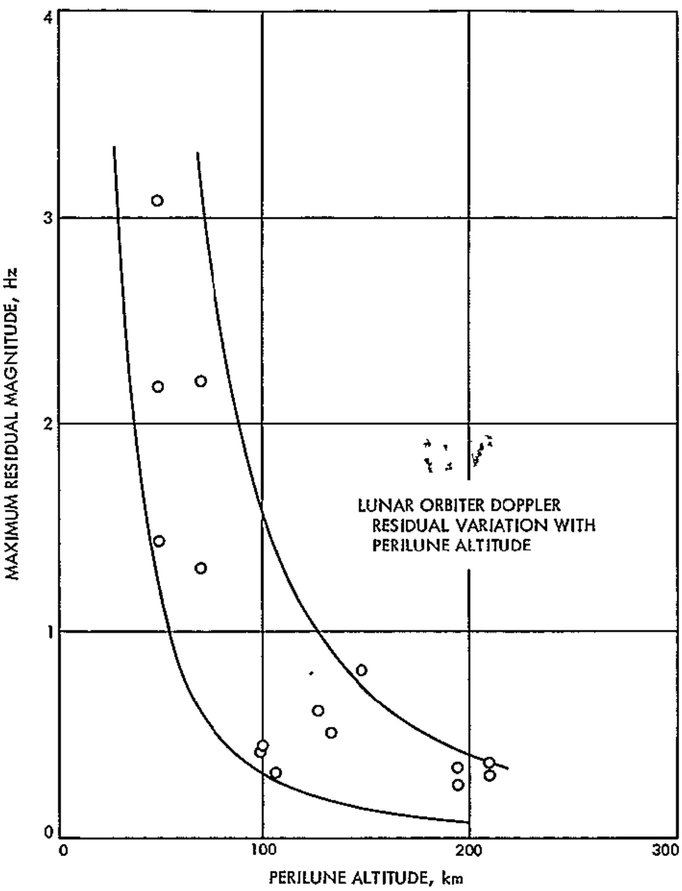

Fig. 2. Effect of perilune altitude on doppler residuals

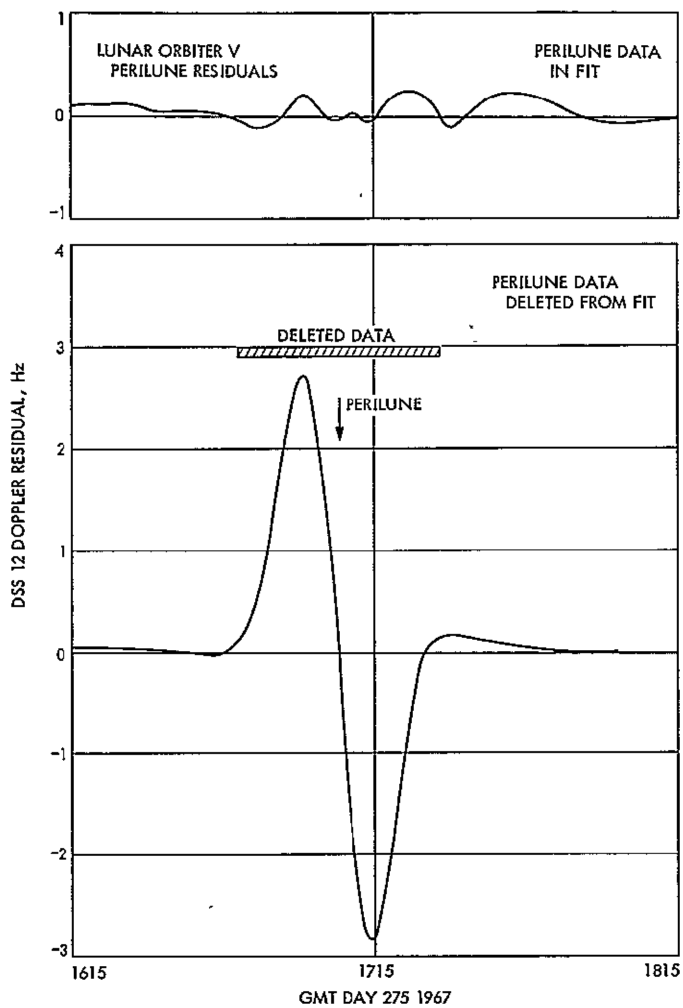

Fig. 3. Effect of perilune data on doppler residuals

The indication that residual magnitude is a function of the inverse square of the perilune altitude is obvious from comparison with the two inverse square curves provided. This phenomenon indicates that perturbing accelerations from lunar surface features could be the cause of the perilune residual problem.

The amount of data included in the arc seriously affects the shape of the perilune residuals. Results of including and deleting data surrounding perilune is shown in Fig. 3 where the second orbit of a three-orbit arc is plotted. A significant difference in the magnitude of the residuals is apparent in the region of perilune. During the first *Lunar Orbiter* mission, doppler data surrounding perilune were deleted from the data arc since the most distinctive residual perturbations occurred in that region. The resulting state vector determinations were used for photography prediction. Most photography was performed at perilune and it has since been determined that the method was of questionable value since the larger doppler residuals indicate a large deviation between the predicted and actual trajectories. A more accurate prediction of photograph locations will be obtained if all data are included in the orbit-determination data arc.

The relative orbit plane-tracking station geometry had a significant effect on the magnitude of the perilune residuals. Figure 4 indicates the residual magnitude is greatest when the orbit plane is edgewise to the line of sight from the tracking stations (longitude of ascending node = 0 deg). Also, there is a slight increase in the frequency of the residual function as the orbit changes position relative to the earth. Since information in the doppler data is only available in the line-of-sight direction (from the tracking station), it is necessary that the residual amplitude exhibit this tendency if surface features are the major cause of the perilune residual problem. Any perturbation in the spacecraft trajectory (in the direction of the lunar surface) when the orbit plane is normal to the line of sight would not be observable in the doppler data, but the same

N69-34635

Status Report of *Lunar Orbiter* Residual Study[1]

Gayle D. Barrow and Philip E. Hong
Space Division, The Boeing Company

I. Introduction

The perilune residual problem first became apparent when the *Lunar Orbiter I* spacecraft was maneuvered into orbit. As the perilune region of the orbit became visible to the earth-based tracking stations, large doppler residuals were found to be present. A doppler residual is simply the difference between the doppler observed by the tracking stations and the doppler computed by the Orbit Determination Program (ODP). To generate a predicted spacecraft ephemeris, the ODP uses a trajectory program containing a specified model intended to describe all significant forces acting on the vehicle, including a spherical harmonic expansion of the lunar gravitational field.

During the translunar portion of the *Lunar Orbiter* mission the doppler residuals were quite small, approaching the noise level of the data as shown in Fig. 1(a). After the spacecraft achieved lunar orbit, the residuals increased by one or two orders of magnitude. The phenomenon is illustrated in Fig. 1(b) for the *Lunar Orbiter IV* mission. The perilune altitude in this case was approximately 2700 km, which accounts for the relatively small residuals.

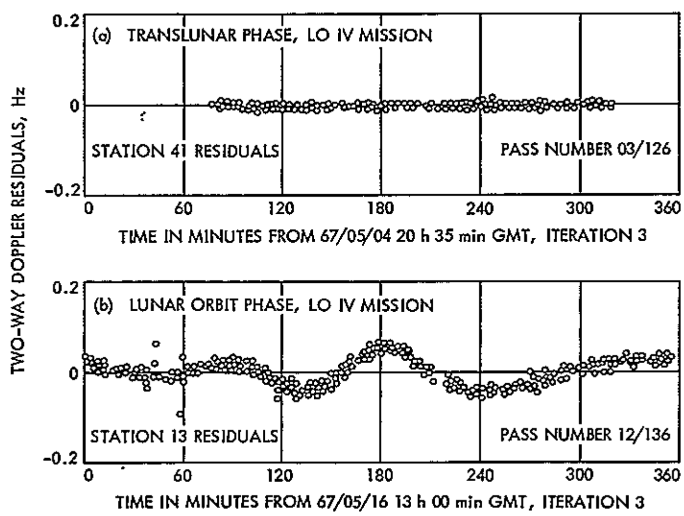

Fig. 1. *Lunar Orbiter* doppler residuals

The relationship of residual magnitude to perilune altitude is shown in Fig. 2. All points are data from *Lunar Orbiter* missions which include various orbital inclinations, apolune altitudes, and tracking station-orbit viewing geometries.

[1]This study was supported by the National Aeronautics and Space Administration, under Contract No. NAS 1-7954.

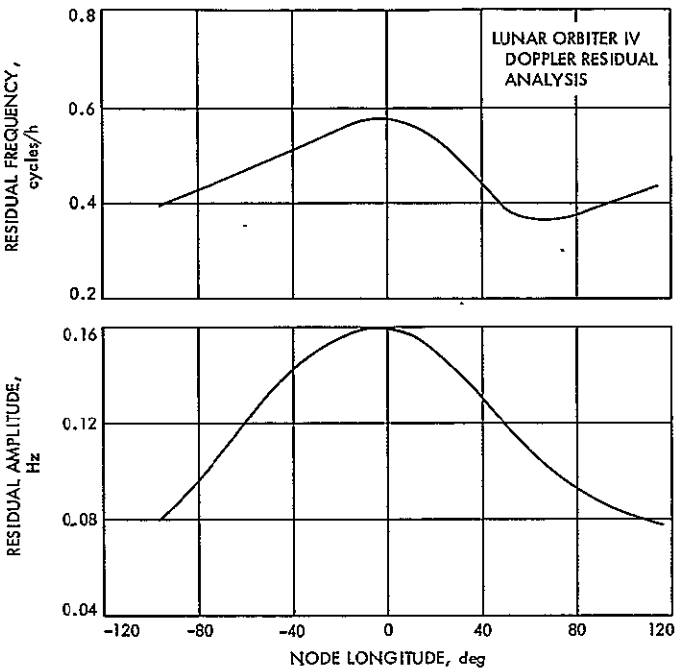

Fig. 4. Effect of orbit plane orientation on doppler residuals

perturbation with the orbit plane parallel to the line of sight would be completely observable.

II. Fourier Analysis

The systematic nature of the doppler residuals during lunar orbit lend themselves to a Fourier analysis. A comparison of the Fourier signatures for the predicted and actual doppler data was performed at both perilune and apolune. The object of this study was to determine whether the doppler data surrounding perilune contains, in its Fourier transform, a dominant spike in a narrow band of frequencies which could be correlated to missing harmonics in the expansion of the lunar potential field. Characteristics of the orbit under consideration are specified in Table 1.

This orbit configuration was selected primarily because the high apolune and low perilune altitudes gave almost no residuals at apolune and relatively large residuals at perilune. The simulated data used for comparison were generated with the ODP. Both a spherical moon and a fourth-order model of the lunar potential were used for computation of the simulated data. There was no significant difference between the Fourier signatures of these data. Figure 5 is a plot of the Fourier coefficients resulting from the analysis of 32 min of doppler data surrounding apolune. There is complete agreement between the coefficients for the simulated and observed data above the noise level of the data (0.01 Hz). Analysis of 32 min of data at perilune resulted in significant differences between the coefficients, as shown in Fig. 6. There is no one dominant coefficient or band of coefficients in either the simulated or observed data of sufficient magnitude to account for the residuals shown.

Table 1. Orbit description, LO IV modified orbit

Epoch	1967 June 16 19^h 00^m 00^s GMT
Apolune altitude	3956.9 km
Perilune altitude	69.6 km
Inclination	85.3 deg
Nodal longitude	334.0 deg
Argument of periapsis	356.0 deg
Period	346.6 min
Time of perilune passage	1967 June 16 19^h 47^m 31.3^s GMT

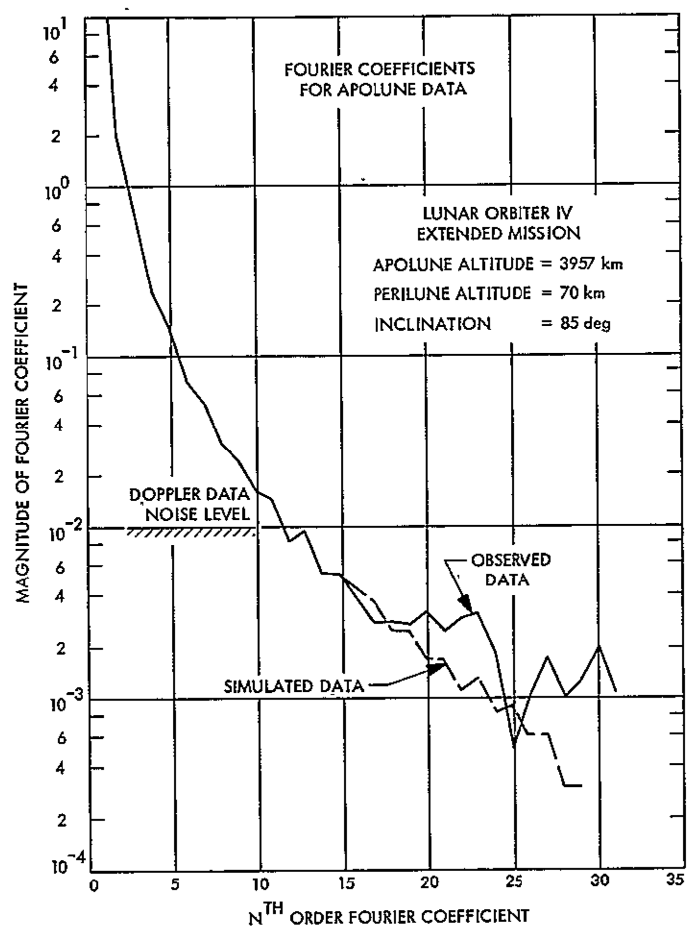

Fig. 5. Fourier analysis of doppler data near apolune

III. One-Way Doppler Tracking

During the *Lunar Orbiter V* extended mission phase, a one-way tracking experiment was conducted to eliminate uplink electronics and multipath effects as a possible cause

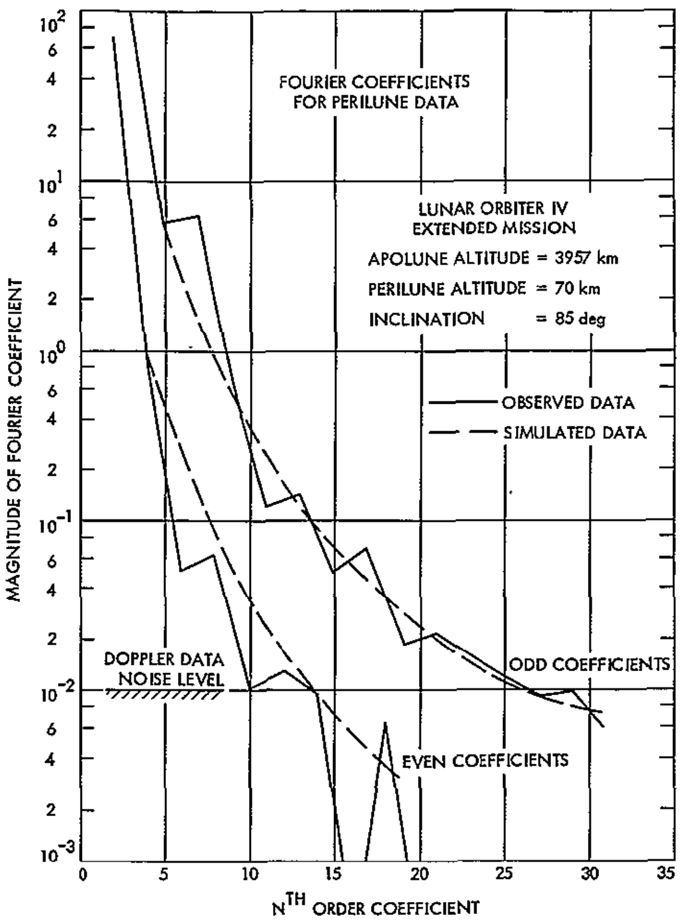

Fig. 6. Fourier analysis of doppler data near perilune

of the residual phenomenon. An oscillator onboard the spacecraft was used as the frequency reference for the doppler tracking. This is in comparison to the normal two-way doppler tracking where the frequency reference is at the tracking station and the spacecraft acts mainly as a signal reflector. One-way doppler residuals during perilune passage are shown in Fig. 7. The high-noise level is due to the inaccuracy of the frequency standard onboard the spacecraft. For comparison, a plot of two-way doppler residuals occurring in the tracking data two orbits prior to the one-way period is also included. Similar to two-way data, one-way doppler residuals approach zero at the apolune of the orbit as shown in Fig. 8. This study, in effect, removes as a possible cause of perilune doppler residuals the station and spacecraft uplink electronics and also the uplink multipath effect (interference of reflected and direct signals to the spacecraft).

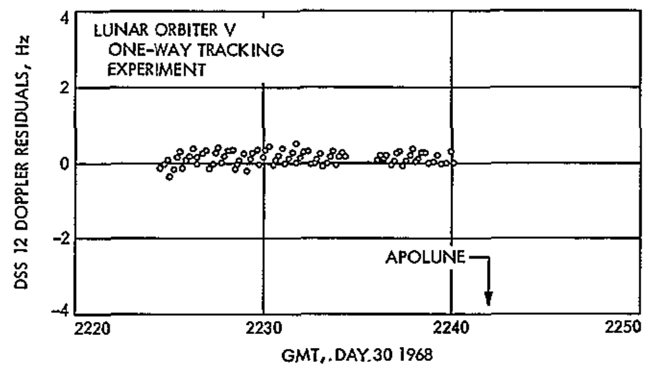

Fig. 8. Doppler residuals of one-way tracking data near apolune

IV. Surface Terrain-Residual Correlation

To investigate the possible correlation between the doppler residuals and the surface terrain below the spacecraft, residuals of different *Lunar Orbiters* were compared as they traversed similar ground tracks. Figure 9 is a plot of

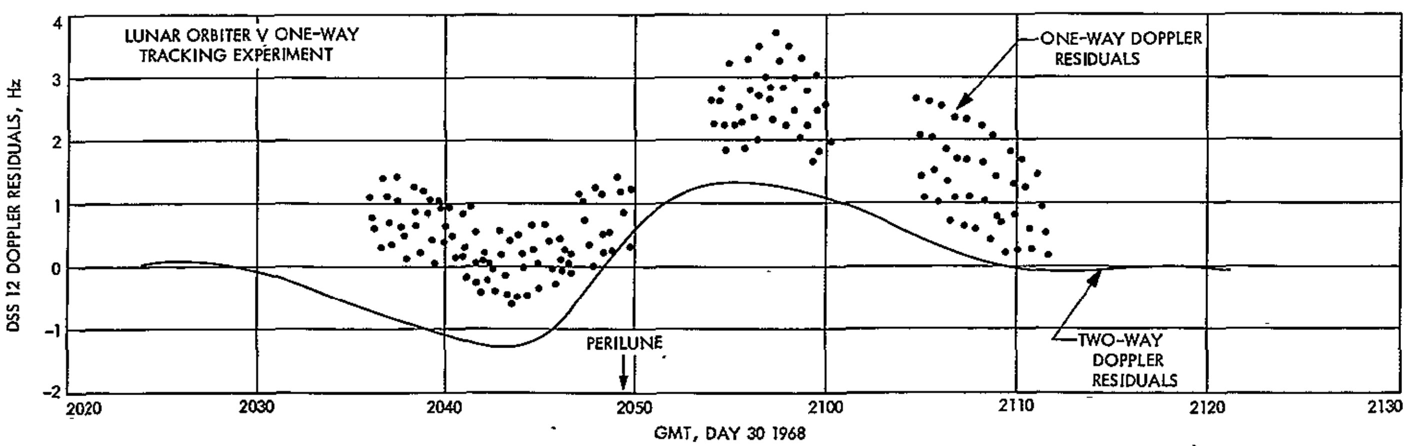

Fig. 7. Doppler residuals of one- and two-way tracking data near perilune

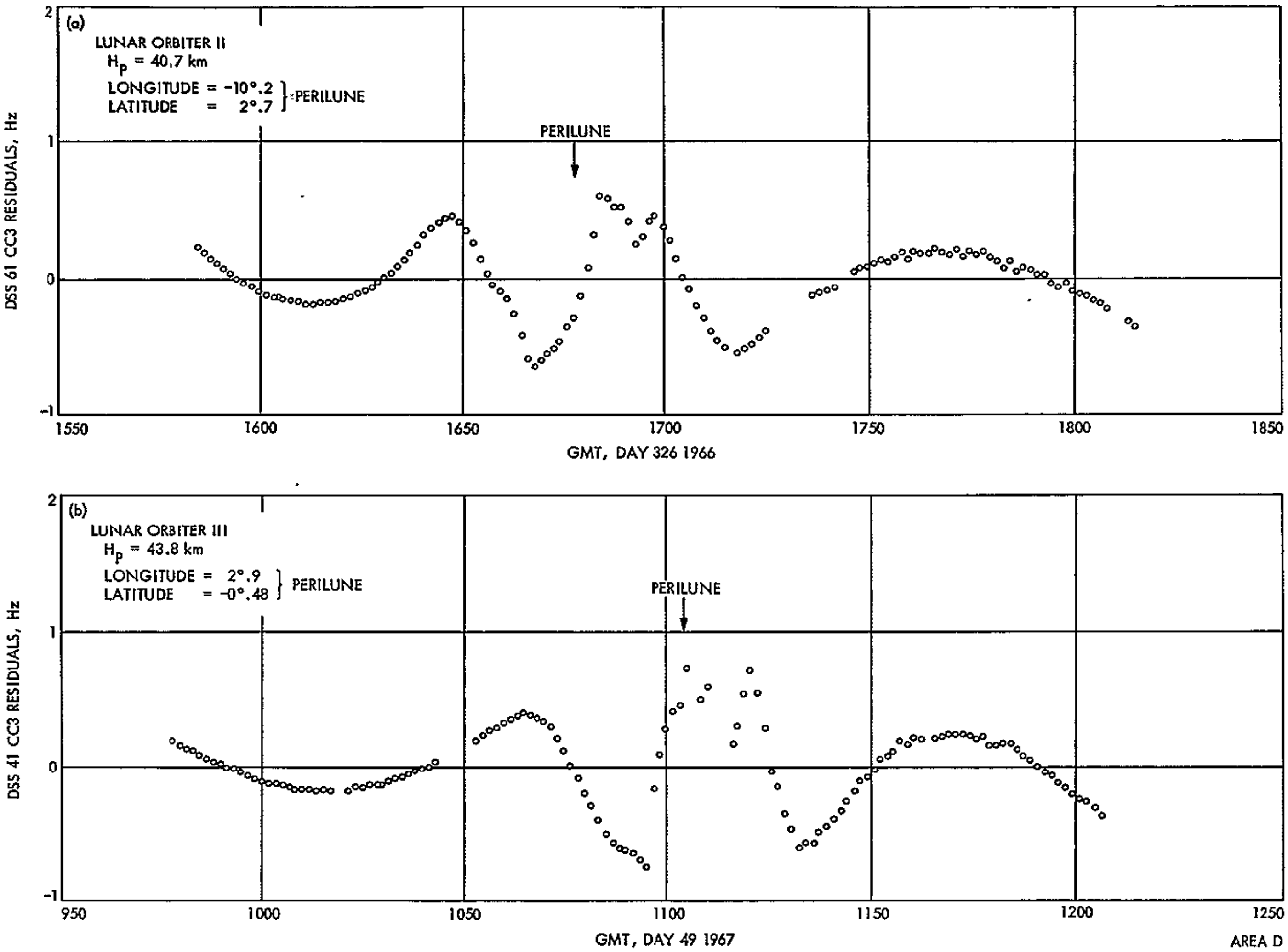

Fig. 9. Doppler residuals over highland region

doppler residuals from *Lunar Orbiters II* and *III* as they pass over the same area. The residuals are formed by the ODP solving for state only with a triaxial moon model. The data arc contains a single orbit of tracking data. Very definite similarities in the residual patterns exist in the region of perilune. Figure 10 is a similar plot with the exception that the two spacecraft are passing over a lunar area which is significantly different from the previous plot. Again, the two *Lunar Orbiters* exhibit definite similarities in their residual patterns, but significant differences exist between Figs. 9 and 10.

The *Lunar Orbiter III* Apollo-type orbit (perilune altitude = 130 km, apolune altitude = 325 km) was investigated for surface correlation. Figure 11 indicates the residual pattern exhibited by the doppler data as the spacecraft passes over the surface terrain shown. Sharp spikes in the residuals are present when there is a drastic change in surface elevation beneath the spacecraft. Also, the oscillations in the residuals correspond closely to the terrain changes. From the investigation, there seems to be a correlation between the residual pattern and the lunar terrain over which the spacecraft is passing.

V. Lunar Atmosphere Investigation

The possibility that the presence of a residual atmosphere due to a cometary impact could contribute to the residual phenomenon was investigated. An experiment conducted with *Lunar Orbiter V* was used to determine the possibility of the existence of a significant atmosphere. The experiment involved the recording of successive spacecraft sun occultation times. Correcting these data for the rotation of the orbit plane with respect to the sunline,

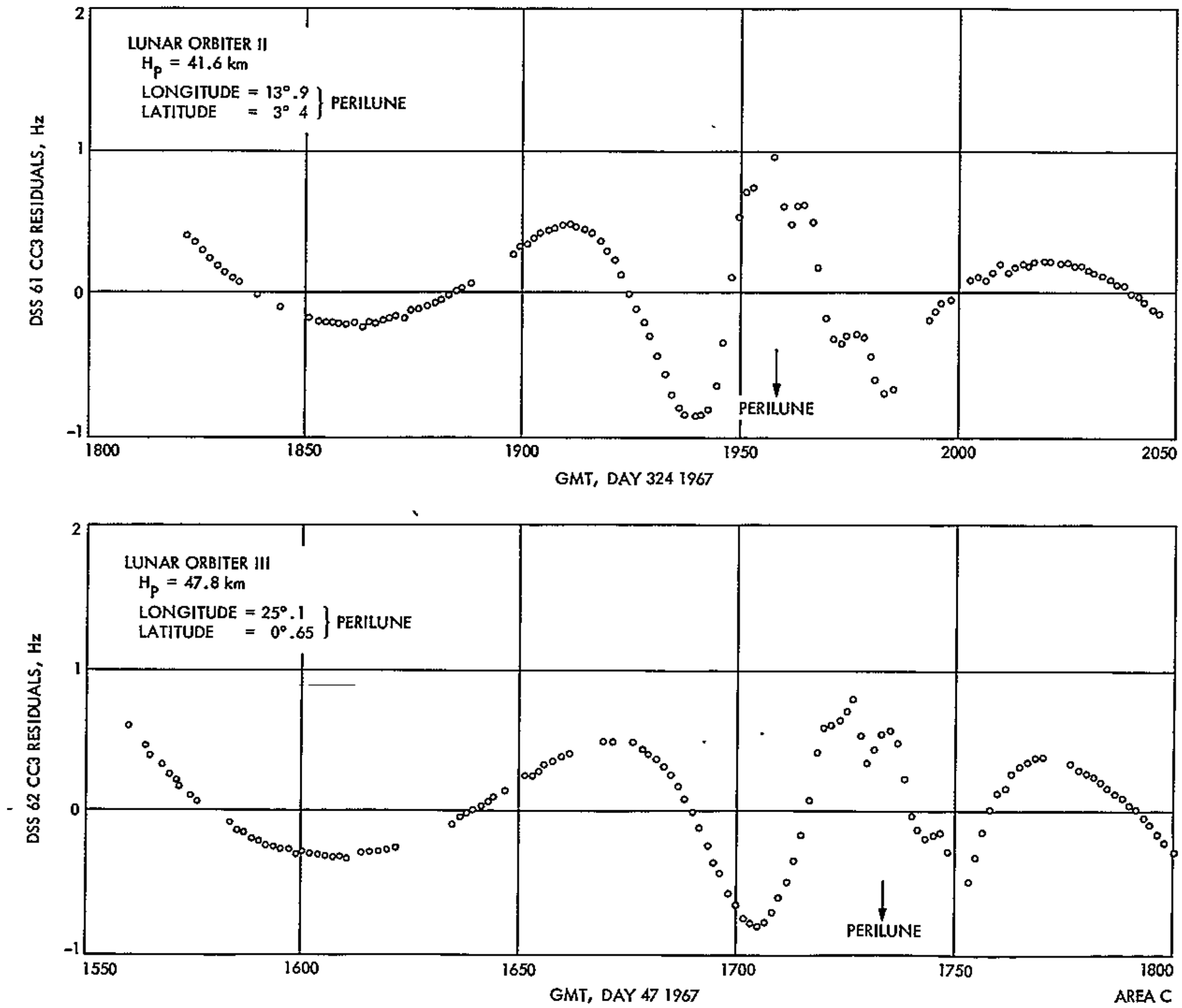

Fig. 10. Doppler residuals over lowland region

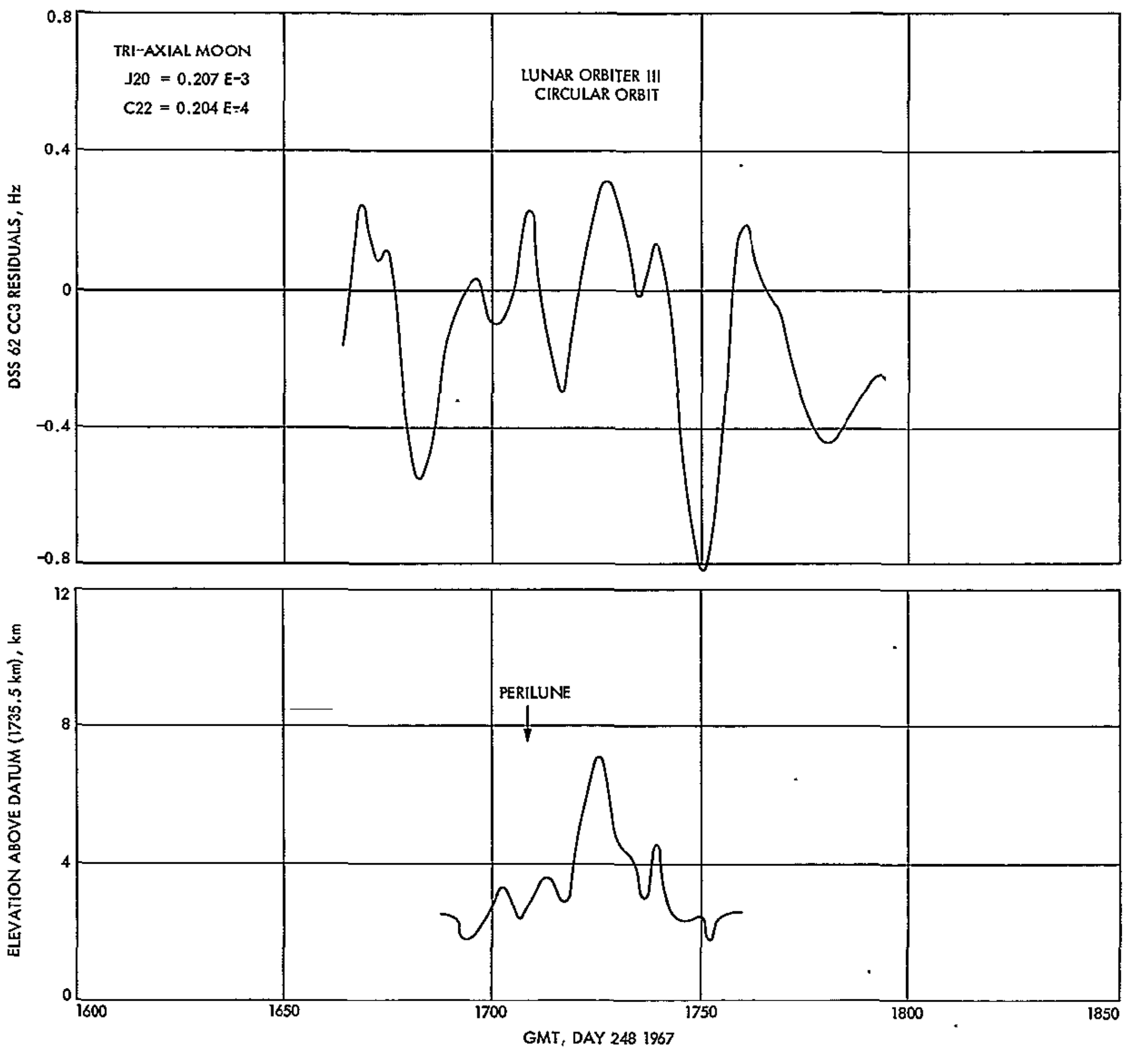

Fig. 11. Comparison of doppler residuals with surface terrain

it is possible to directly measure the orbital period of the spacecraft. For the 40-day period shown in Fig. 12, no secular decay in the orbital period is obvious, although a long-term oscillation occurs, possibly due to occultation over areas of differing elevation. This result indicates that, at an altitude of 100 km (perilune altitude of the orbit), no significant lunar atmosphere can exist.

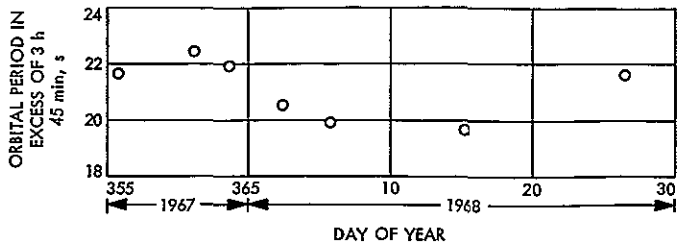

Fig. 12. Orbital period computed from sun occultation data

VI. Summary

As a result of this study, it can be concluded that there is a distinct correlation between the doppler residual pattern and the surface terrain beneath the spacecraft.

Investigation of the simulated and actual doppler tracking data, using Fourier transform methods, shows a distinct difference between the signatures of the simulated and actual data, but no dominant coefficients exist in any frequency range which could consistently account for the residuals.

One-way doppler tracking data exhibit similar residual patterns to that of two- and three-way doppler data, thus verifying that no problem exists in the uplink electronic systems.

Direct measurement of the orbital period of a *Lunar Orbiter* indicates that the existence of a significant lunar atmosphere above the altitude of 100 km is very unlikely.

There is a very serious need for more analysis on the lunar gravitational model, as can be seen from the magnitude of the doppler residuals experienced in this study.

Discussion

Kopal: Any atmospheric effect would vary inversely with the altitude of periselenium. Have you performed an analysis of any of the satellites with periselenia below 50 km?

Barrow: No, those low satellites were corrupted by maneuvers, whereas we were relatively free of that in the *Apollo*-type orbits.

Kopal: Have you used these data to infer an upper limit to the density?

Barrow: No, although I think that, based on other data, we once obtained an upper limit of about 10^{-12} slug/ft^3.

Kopal: The optical upper limit is about 10^{-10} gm/cm^3; anything greater could be detected by optical methods from earth. Because of knowledge of the solar wind, we can say that the density of the transient exosphere around the moon must be at least 10^{-31}gm/cm^3. Between these two values is a no-man's land.

Sconzo: I believe that another point is the most important one, that of knowledge of the gravitational model of the moon, because, if you compare various results, you do not find agreement even for J_2.

N69-34636

A Lunar Mass Distribution

Paul M. Muller and William L. Sjogren
Jet Propulsion Laboratory, Pasadena, California

Abstract[1]

Lunar Orbiter *tracking data have been processed to give a qualitatively consistent gravimetric map of the lunar nearside. While a simplified model was employed, the results indicate that there are large mass concentrations under the lunar ringed maria. These mass concentrations may have important implications for the various theories regarding the evolution of the moon.*

Discussion

Question (unidentified): How does the anomaly on Mare Oriental compare to the others in size?

Muller: Because of the geometry, it is rather difficult to tell. The table of measured values is given in the *Science* article (Ref. 1). We feel that we located the center to within approximately 2 deg, but one has to make a geometric assumption. We get a magnification of approximately an order of magnitude from the raw data to the inferred accelerations. It is not well enough defined for us to make a statement, but it is certainly large—surprisingly so. Mare Oriental is much smaller than Mare Imbrium, but the mascon may be nearly as large.

Kopal: Two of your *Surveyors* landed in your gravitational anomalies and a third landed far away. Chemical analysis at the landing sites in Mare Tranquilitatis and Sinus Medii showed more iron than did the landing near Tycho. It may well be that, as previously conjectured, the surface material is more dense in these areas. I think that we should be very careful not to jump to conclusions too fast. We do not have to bury the heavy material too far below the surface, since the lava covering these maria may be of higher density than the surrounding material. How far would this go toward explaining the positive anomalies? If the heavy material is too low below the surface, there are many difficulties. First, you cannot keep it suspended there for an astronomically long time. Second, you will need too heavy material if you put it too far below the surface. If you bring it near the surface, you will not need such a high density. Another question that presents itself: If you have a high-density region in a mare, and if you have a "hard lander" spacecraft impact in this part, the heavy material would exert an increased attraction, would accelerate the spacecraft. The inferred radius at impact would look like an excess radius, i.e., the spacecraft would impact sooner than expected. There was a hard landing in Mare Imbrium (Lunik 2) for which the data are fully published. Did they find an excess radius? Does this excess correlate in any way with your data?

Sjogren: We anticipate this possibility, and we shall certainly look for it when we are able to handle the data in a more quantitative way.

[1]Abstract only (see Refs. 1 and 2 for a complete discussion).

Discussion (contd)

Kopal: You are well aware that some of the Russian orbiters carried magnetometers. If the excess density is caused by iron-rich material, then they should have been observing magnetic anomalies when their spacecraft overflew these regions. So far, they have revealed no such results. It might be that they observed something consistent with your findings and were unable to understand it properly. This may help them.

Bender: The high concentration of magnetic materials will not necessarily give a magnetic anomaly unless there is some mechanism to align them. This requires the presence of a field to start with, which there does not seem to be.

Kopal: The existence of a concentration of iron-rich material is not a sufficient, but a necessary condition.

Sconzo: Can these results be used to determine the lunar potential?

Muller: Yes! Jack Lorell at JPL has already begun processing these results into harmonic expansions, although the low-order terms will be missing. He already has some pretty fifteenth-order expansions. We feel that, once the high-frequency terms are determined in this way, satisfactory fits can be obtained over arcs of several revolutions. *Then,* we think the previously-tried techniques can be used for the low-order terms.

Lyttleton: If, instead of concentrating the mass in a point, you spread it out in a annulus, how far could you carry this without affecting your results?

Muller: It is a distinct possibility that the mass is more diffuse than our present results indicate. The qualitative aspect of our results is very good, but the quantitative aspect is not. Some of the mass *could* be spread around. I rather doubt that the bulk of the mass could exist as a small density increase over the entire ringed sea.

Lyttleton: What about a distribution entirely around the ring?

Muller: No, that is difficult to reconcile with the rate of change of the accelerations. If it were really a ring of extremely high density, we would expect to see two peaks in the spacecraft accelerations, because it is only 150–200 km above the surface and the mare may be on the order of 1000 km. If it were a ring, I should think that we would resolve it quite easily. Our resolution is not 10 km, but is certainly 100–150 km. Similarly, if the mass were completely diffuse, we would expect to see a much flatter acceleration curve than we do. Now it is true that, if the mass were spherical, it would look like a point-mass located somewhere below the surface, but it could hardly be a 500-km sphere centered 100 km below the surface. While our results are preliminary, I think that they require the masses to be relatively small, relatively high-density variations, or at least include some such component. It may be true that the surface material in these maria are iron-rich, but in conjunction with the lump in the center.

Kopal: This location of a small, heavy object 100 km below the surface involves some mechanical difficulties. The density must exceed that of the surrounding medium by at least a factor of 2. No matter how you put it there, it cannot remain for an astronomically long time, even if the moon has no liquid core. The moon is apparently cold, but such a heavy object would, in 10^9 years, settle even through solid rocks. So these planetesimals should be found near the center of the moon, not suspended near the surface. I fear that we will hear that these phenomena must have occurred in the recent past, but if this were true, the scars of similar events should still be visible on earth. I think none have been found so far.

Unidentified: We have many gravity anomalies on earth that are due to tectonics. I do not think that your results are necessarily a proof of impact.

Muller: No, sir, they do not! I only noted that these surface features are widely mentioned in the literature as impact features. I certainly will not rule out the possibility of other explanations.

Hansen: Even if Lorell succeeds in constructing a spherical harmonic potential field, it will be limited. What is the reason for doing it? Why get such a model?

Sjogren: We are not particular, but that is the form people want it in.

Unidentified: Back to the settling of these masses, the Mesabi Range was a field very rich in nickel-iron yet was very close to the surface, in a much more fluid body.

Kopal: Relative to the size of the earth, this was a very tiny disturbance. I doubt that the existence of the Mesabi Range was ever detected by earth satellites.

Unidentified: Well, of course, it had been mined for many years before.

Muller: If Urey's idea is correct, then what happened is that there was a cauldron of molten material, out of which the nickel-iron and heavy silicates were processed.

Kopal: The argument is credible, but it all depends on the partition of energy from the impact. How much of the impact energy will be converted into melting? It is most likely quite small.

Muller: It depends on the speed of impact, and this is argued by geophysicists.

N69-34637

The Shape of the Moon as a Surface of Helmert's Type

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We consider the function U of the external potential of a planet expanded in zonal, sectorial, and tesseral harmonics up to terms of fourth degree and fourth order. Denoting by U_4 this truncated expression for U we can write

$$U_4 = k\left[\frac{1}{r} + \frac{a^2}{r^3}\{A_{2,0}Y_{2,0} + A_{2,2}Y_{2,2} + B_{2,2}Z_{2,2}\} + \frac{a^3}{r^4}\left\{A_{3,0}Y_{3,0} + \sum_{m=1}^{3}(A_{3,m}Y_{3,m} + B_{3,m}Z_{3,m})\right\} + \frac{a^4}{r^5}\left\{A_{4,0}Y_{4,0} + \sum_{m=1}^{4}(A_{4,m}Y_{4,m} + B_{4,m}Z_{4,m})\right\}\right] \tag{1}$$

where

$k =$ the gravitational constant

$a =$ the equatorial radius of the planet

$r =$ the radial distance

$A_{n,m}, B_{n,m} =$ numerical coefficients proportional to the harmonic coefficients $C_{n,m}, S_{n,m}$, respectively

and $Y_{n,m}, Z_{n,m}$ are the functions defined as follows:

$$Y_{n,m} = X_{n,m}\cos m\lambda$$

$$Z_{n,m} = X_{n,m}\sin m\lambda$$

$$X_{n,m} = \frac{1}{2^n(n+m)!}(1-t^2)^{m/2}\frac{d^{n+m}(t^2-1)^n}{dt^{n+m}}$$

$t = \cos\theta$

$\theta, \lambda =$ polar coordinates, respectively, colatitude and longitude

The relationship of proportionality between $A_{n,m}$ and $C_{n,m}$ is

$$A_{n,m} = \frac{(n+m)!}{n!}C_{n,m} \tag{2}$$

and a similar relationship holds between $B_{n,m}$ and $S_{n,m}$.

A surface of equilibrium of Helmert's type (Ref. 3) is defined by the equation

$$kH + \frac{1}{2}\omega^2 r^2 \sin^2\theta = \text{constant}$$

where H is the Helmert function which contains the term $1/r$ and the terms in $A_{2,0}$, $A_{2,2}$, $A_{4,0}$, $A_{4,2}$ and $A_{4,4}$ of U_4, and ω is the angular velocity of the planet around its rotation axis.

Transforming the polar coordinates r, θ, λ into Cartesian coordinates x, y, z, it is easily seen that Eq. (2) becomes an algebraic equation of 22nd degree. It is the equation of a closed surface which "resembles" the surface of a sphere. When the coordinates, r, θ, λ are taken at points of the planetary surface S we should have

$$kH_{(s)} + \frac{1}{2}\omega^2 \{r \sin\theta\}^2_{(s)} = \text{constant}$$

or

$$H_{(s)} = K - \frac{\omega^2}{2k} r^2_{(s)} \sin^2\theta \qquad (3)$$

where K is a constant. In writing Eq. (3), we have assumed that S is also a surface of equilibrium.

We want to approximate this quasi-spherical surface with that of a triaxial ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \qquad (a > b > c) \qquad (4)$$

It can be seen that the radial distance in a triaxial ellipsoid can be expanded in a convergent series as follows

$$r = a\,[1 + \alpha_{0,0} Y_{0,0} + \alpha_{1,0} Y_{2,0} + \alpha_{1,1} Y_{2,2} + \alpha_{2,0} Y_{4,0} + \alpha_{2,1} Y_{4,2} + \alpha_{2,2} Y_{4,4} + \cdots] \qquad (5)$$

where the coefficients $\alpha_{n,m}$ are, in turn, power series in two geometric parameters e^2, ϵ^2 defined as follows

$$e^2 = \frac{a^2 - c^2}{a^2}, \qquad \epsilon^2 = \frac{a^2 - b^2}{a^2} \qquad (6)$$

The quantities e and ϵ are the eccentricities of two ellipses. The first ellipse is the intersection of Eq. (4) with the prime meridian plane xz, while the second ellipse is the intersection of the same surface with the equatorial plane xy.

The explicit expressions for the series $\alpha_{n,m}$ have previously been obtained by the author in his investigation on the triaxiality of the earth surface (Ref. 4).

The evaluation of $H_{(s)}$ at points of the surface (Eq. 4) is a very complex algebraic task more cumbersome than when it is assumed that $a = b$ (ellipsoid of revolution). It is not only necessary to find the expansions for the following powers of the radial distance r^{-1}, r^{-3}, and r^{-5}, but it is also necessary to get rid of the many mixed products among the Y functions. Fortunately, it has been proved (Ref. 5) that any product of the kind $Y_{n,m} \cdot Y_{n',m'}$, is a linear combination of Y functions with constant coefficients. Thus, after finding for any mixed product of interest the corresponding set of constant coefficients, it can be shown that the end result of the evaluation is of the following form:

$$H_{(s)} = H_{0,0} + H_{1,0} Y_{2,0} + H_{1,1} Y_{2,2} + H_{2,0} Y_{4,0} + H_{2,1} Y_{4,2} + H_{2,2} Y_{4,4} + \cdots \qquad (7)$$

The various components $H_{n,m}$ of $H_{(s)}$ are also known series in e^2, ϵ^2 (Ref. 2). The algebraic manipulations required to arrive at Eq. (7) have been performed first by hand to a lower order and then checked and extended automatically by a computer to higher order powers in e^2, ϵ^2, using for this purpose the IBM FORMAC language. As an example, we present here the explicit expression for $H_{2,2}$

$$\begin{aligned} H_{2,2} = \frac{1}{a}\Bigg[1 &+ \frac{5}{22}A_{4,4}e^2 + \left(-\frac{3}{2}A_{2,2} + \frac{10}{11}A_{4,2} + \frac{25}{22}A_{4,4}\right)\epsilon^2 + \frac{305}{1144}A_{4,4}e^4 \\ &+ \left(-\frac{3}{44}A_{2,2} - \frac{30}{143}A_{4,2} + \frac{75}{572}A_{4,4}\right)e^2\epsilon^2 \\ &+ \left(-\frac{1}{4} - \frac{3}{11}A_{2,0} - \frac{81}{44}A_{2,2} + \frac{135}{286}A_{4,0} + \frac{485}{286}A_{4,2} + \frac{1975}{1144}A_{4,4}\right)\epsilon^4\Bigg] + O(e^6, \epsilon^6) \end{aligned}$$

The second term of the right-hand side of Eq. (3) requires the knowledge of the expression for $r^2_{(s)}$ and also the execution of the product $-r^2_{(s)}\sin^2\theta$ which can obviously be written as follows:

$$-r^2_{(s)}\sin^2\theta = \frac{2}{3}r^2_{(s)}(Y_{2,0}-1) =$$

$$\frac{2}{3}a^2(-\beta_{0,0}+\beta_{1,0}Y_{2,0}+\beta_{1,1}Y_{2,2}+\beta_{2,0}Y_{4,0}+\cdots)$$

Any coefficient $\beta_{n,m}$ is again a power series in e^2, ϵ^2. As an example, the explicit expression for $\beta_{0,0}$ is

$$\beta_{0,0} = 1-\frac{1}{5}e^2-\frac{2}{5}\epsilon^2-\frac{4}{35}e^4-\frac{4}{35}e^2\epsilon^2-\frac{1}{7}\epsilon^4+O(e^6,\epsilon^6)$$

Finally, to find the relationships among the geometrical parameters e, ϵ and the numerical constants $A_{n,m}$, we consider Eq. (3) and, by equating the coefficients corresponding to the same Y function on both sides, we obtain

$$\left.\begin{aligned} K &= H_{0,0}+\sigma\beta_{0,0} \\ H_{n,m} &= \sigma\beta_{n,m} \end{aligned}\right\} \tag{8}$$

where

$$\sigma = \frac{1}{3}\frac{\omega^2a^3}{k} \tag{9}$$

Leaving momentarily aside the first equation in Eq. (8), each other equation contains the various powers of e and ϵ and, linearly, the coefficients $A_{n,m}$ intermingle each other. If we assume that the $A_{n,m}$ coefficients are experimentally known, then these equations can be linearized and two corrections de and $d\epsilon$ determined. These corrections added to two estimated values $\overset{*}{e}$ and $\overset{*}{\epsilon}$

$$e = \overset{*}{e}+de, \qquad \epsilon = \overset{*}{\epsilon}+d\epsilon$$

will provide the improved values of e and ϵ.

Any linearized equation assumes the following form

$$\overset{*}{e}\,[F^{(1,0)}_{n,m}+2\overset{*}{e}^2F^{(2,0)}_{n,m}+\overset{*}{\epsilon}F^{(1,1)}_{n,m}]\,de$$
$$+\overset{*}{\epsilon}\,[F^{(0,1)}_{n,m}+2\overset{*}{\epsilon}^2F^{(0,2)}_{n,m}+\overset{*}{e}^2F^{(1,1)}_{n,m}]\,d\epsilon = -\frac{1}{2}F_{n,m}(\sigma,\overset{*}{e},\overset{*}{\epsilon}) \tag{10}$$

where $F_{n,m}(\sigma,\overset{*}{e},\overset{*}{\epsilon})$ is a constant. This constant is specified to be

$$F_{n,m} = F^{(0,0)}_{n,m}+F^{(1,0)}_{n,m}\overset{*}{e}^2+F^{(0,1)}_{n,m}\overset{*}{\epsilon}^2+F^{(2,0)}_{n,m}\overset{*}{e}^4$$
$$+F^{(1,1)}_{n,m}\overset{*}{e}^2\overset{*}{\epsilon}^2+F^{(0,2)}_{n,m}\overset{*}{\epsilon}^4$$

where the coefficients $F^{(r,s)}_{n,m}$ are also constants and known expressions in σ and the various $A_{n,m}$. As an example, we give the explicit expressions for the case $n=2, m=1$

$$F^{(0,0)}_{2,1} = A_{4,2}$$

$$F^{(1,0)}_{2,1} = \frac{3}{14}A_{2,2}+\frac{155}{154}A_{4,2}$$

$$F^{(0,1)}_{2,1} = -\frac{9}{14}A_{2,0}-\frac{3}{28}A_{2,2}+\frac{75}{77}A_{4,0}+\frac{115}{154}A_{4,2}+\frac{5}{154}A_{4,4}-\frac{3}{7}\sigma$$

$$F^{(2,0)}_{2,1} = \frac{75}{308}A_{2,2}+\frac{11615}{8008}A_{4,2}$$

$$F^{(1,1)}_{2,1} = \frac{1}{14}-\frac{3}{44}A_{2,0}+\frac{15}{616}A_{2,2}+\frac{45}{143}A_{4,0}+\frac{1245}{4004}A_{4,2}-\frac{15}{2002}A_{4,4}-\frac{30}{77}\sigma$$

$$F^{(0,2)}_{2,1} = -\frac{1}{28}-\frac{237}{308}A_{2,0}-\frac{87}{616}A_{2,2}+\frac{885}{572}A_{4,0}+\frac{8815}{8008}A_{4,2}+\frac{485}{8008}A_{4,4}+\frac{15}{77}\sigma$$

The system of linearized equations, such as Eq. (10), can be solved by an iterative procedure using the method of least squares. Then, taking into account the first equation in Eq. (8), we will also obtain the value of the constant K which is consistent with the assumption made above that Eq. (3) represents an equipotential surface.

As a result of the approximation achieved by least squares and the fact that we have neglected some terms of U_4 in considering the function H, the hypothetical surface of equilibrium

$$U_4 + \frac{1}{2}\omega^2(x^2 + y^2) = \text{constant}$$

will exhibit small undulations when compared against the surface of the triaxial ellipsoid.

It should be mentioned that the formulation developed in this paper is a rearrangement and generalization of an earlier formulation originated by Somigliana (Ref. 5) who considered the triaxial ellipsoid approximating a surface of equilibrium of Bruns type (Ref. 7). Such a surface is a particular case of the Helmert surface, namely, that in which the function H contains only the two coefficients $A_{2,0}$ and $A_{2,2}$. The degree of the equation of the Bruns surface reduces to the fourteenth degree.

The results of our formulation, when applied to the moon, are presented in Table 1. These results were obtained on the basis of two different models of the potential function as determined by scientists at JPL and the Soviet Union, respectively. The computation was carried out in canonical units from an assumed value of "a" as indicated and referenced in Table 1. The discrepancies among the two sets may be attributed to the fact that the coefficients $A_{4,2}$ and $A_{4,4}$ corresponding to the JPL model might be unreliable.

Table 1. Ellipsoidal figure of the moon

Symbols	Potential model[a]	
	JPL (Ref. 9)	Russian (Ref. 10)
e	0.03829 ± 0.01637	0.038226 ± 0.000394
ϵ	0.01635 ± 0.01424	0.012967 ± 0.000423
$f_e = 1 - \sqrt{1 - e^2}$	$0.00073321 = \frac{1}{1363.9}$	$0.00073087 = \frac{1}{1368.2}$
$c = a\sqrt{1 - e^2}$	1736.6 ± 1.1 km	1736.6 ± 0.03 km
$f_\epsilon = 1 - \sqrt{1 - \epsilon^2}$	0.00013366	0.00008407
$b = a\sqrt{1 - \epsilon^2}$	1737.7 ± 0.4 km	1737.8 ± 0.01 km
K	1.00054235	1.00052505 Note: In this model, the coefficients $A_{4,2}$ and $A_{4,4}$ are unknown.

[a]Assumed value (Ref. 8) of $a = 1737.90$ km
Canonical unit of time $= 1035.739$ s
Value of $\sigma = 0.000253340$.

Discussion

Sjogren: The Russians only published a third order set of harmonics, didn't they?

Sconzo: No, they also used fourth order.

Eichhorn: Isn't what you are doing taking the moon and fitting the best tri-axial ellipsoid?

Sconzo: Yes, you consider the potential of the moon, plus the centrifugal force, and you obtain a surface of degree 22. Then you try to fit an ellipsoid to it.

Lyttleton: In other words, you are applying an assumption of a homogeneous moon?

Sconzo: Of course, I am using the expression for the external potential on the surface.

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AFCRL Computer Programs for the Physical Ephemeris of the Moon

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The purpose of my presentation is to informally and briefly describe a computer technique used at the Air Force Cambridge Research Laboratories (AFCRL) for the analytic generation of tables of the physical ephemeris of the moon and to outline three computer programs which use these tables to provide data relevant to the observation and reduction of observations of the librations of the moon.

A Cartesian coordinate system for the moon is defined whose axes are assumed to be the same as those of the moon's principal moments of inertia. Axis 1 is the mean direction of the earth, Axis 3 is the direction of the north lunar pole, and Axis 2 completes an orthogonal right-handed system. Let ω_1, ω_2, ω_3 be the angular rotational velocities of the moon about its 1, 2, 3 axes, and let u_1, u_2, u_3, be the direction cosines of the earth from the moon. Let r be the earth–moon distance, and a the mean distance. Set $\alpha = (C - B)/A$, $\beta = (C - A)/B$, and $\gamma = (B - A)/C$ where A, B, and C are the moments of inertia about the 1, 2, 3 axes. Then the Euler dynamical equations are

$$\dot{\omega}_1 + \alpha\,\omega_2\omega_3 = \alpha\, k^2\,(a/r)^3\, u_2u_3 + \text{similar solar terms}$$

$$\dot{\omega}_2 - \beta\,\omega_3\omega_1 = -\beta\, k^2\,(a/r)^3\, u_3u_1 + \text{similar solar terms}$$

$$\dot{\omega}_3 + \gamma\,\omega_1\omega_2 = \gamma\, k^2\,(a/r)^3\, u_1u_2 + \text{similar solar terms}$$

If the unit of time is chosen as one tropical month, we have $k^2 = 3 \times 0.9905$. This factor would be exactly 3 if (1) Kepler's third law were exact for the motion of the moon about the earth, (2) the mass of the moon were quite negligible, and (3) the unit of time were the sidereal month.

Symbolically, the Euler equations may be represented by

$$Q\,\Omega = Y$$

where Q represents a nonlinear operator taking

$$\Omega = (\omega_1\,\omega_2\,\omega_3)^T$$

in cross-coupled terms and first-order time derivatives. The term Y represents the right-hand side of the Euler equations. The factor (a/r) is the sine parallax from the lunar theory, but the values of the direction cosines u_i depend on the lunar theory plus the integrals of ω_i.

The integrals of ω_i may be transformed to $X^T = (p_2\,p_1\,\tau)$ where p_i are the direction cosines of the pole of the ecliptic and τ is the perturbation of the moon from uniform polar rotation. The Euler equations are then transformed, symbolically, into

$$RX = Y$$

where R represents a nonlinear operator taking X in cross-coupled terms and time derivatives to the second order; and now $Y = Y(X)$. Because it is nonlinear, R has no unique inverse. A method of solution is found, however, by rewriting the above equation as

$$TX = Y + (T - R)X$$

where T is an operator which has the following properties: (1) it is linear and can be inverted almost everywhere, and (2) it is chosen such that the linearization of $Y + (T - R)X$, which is the right-hand side of this equation, contains no components of X. By formally premultiplying this equation by T^{-1}, we get

$$X = X + T^{-1}(Y - RX)$$

which yields the iterative procedure

$$\Delta X = T^{-1}(Y - Q\Omega)$$

$$X \leftarrow X + \Delta X$$

Where

$$Y = Y(X)$$

$\Omega = \Omega(X)$. The term $Q\Omega$ replaces RX in the calculation because it is a simpler expression in practice. With the use of this iterative technique, where $\Delta X \to 0$, $Y - Q\Omega \to 0$ and a solution of the Euler equations is found.

The Euler equations for the forced physical librations of the moon have been solved for various combinations of β and γ with a digital computer performing all the semi-literal mathematical manipulations of the iterative technique outlined above in the program, LIB. The solutions generated are not unique, but they are stable and correspond to the unique solutions for the forced physical librations of the linearized Euler equations. These solutions are quite adequate for describing the forced physical librations of the moon except under near-resonant conditions which may be excluded from consideration on the basis of astronomical observations.

Table 1 presents the printed output for the particular set of parameters, $\beta = 0.0006268$ and $\alpha = 0.0002300$. This set of parameters was chosen to conform with Koziel's solution (Ref. 11), $f = \alpha/\beta = 0.633$ and I (the mean inclination of the equator of the moon to the ecliptic) $= 5521''$, by using the formula

$$I = -1612'' - 5''.2 \times 10^4 \gamma + 11''.4 \times 10^6 \beta$$

This formula was established from other runs of the basic computer program for various sets of the parameters β and γ. Tabulated in Table 1 are the coefficients of each of the terms of the trigometric series in arc seconds, and the integral coefficients of the Delaunay arguments, respectively, l, l', F, and D, in the arguments of the trigometric terms.

Included in the solution table are the perturbations to the motions according to Cassini's laws: τ, $I\sigma$, and ρ. These are ordinarily used along with the lunar ephemeris, Cassini's laws, and topocentric correction formulae to calculate topocentric librations of the moon. A program, PHILA, was written to do this directly for any observatory and any date and time. This program included the generation of a crude lunar ephemeris (truncating terms less than 2''.0 in longitude and latitude, and 0''.015 in sine parallax). The choice of the parameters β and γ is arbitrary, for the coefficients of τ, $I\sigma$, and ρ are set up in the program as functions of β and γ. This program is convenient if information such as the approximate geocentric and topocentric coordinates of the moon are desired.

Calculation of the librations of the moon by PHILA is rather indirect and much more complicated than necessary if only the librations are required. The sine parallax is required to make topocentric corrections, but the longitude and latitude of the moon are only required explicitly to calculate the position angle of the moon. The position angle C is referred to the lunar meridian—the great circle passing through the center of the moon and the pole of the earth on the celestial sphere. It can be broken into two parts: (1) the analogous angle H referred to the great circle passing through the center of the moon and the pole of the ecliptic, and (2) the angle C–H between this great circle and the meridian. The angle C–H depends only on the lunar theory and does not depend on the rotations of the moon. The angle H, however, does depend on the rotations of the moon and is, therefore, a function of the libration parameters. It may be easily calculated from the vectors u_i and p_i (for H', the topocentric angle, use u_i' and p_i). PHILB is a program parallel to PHILA which was written to calculate the librations of the moon (including H and H' instead of C and C') directly from the expansions for the sine parallax, u_i and p_i.

Table 1. Computer listing for AFCRL lunar librations

5051E1 ECKHARDT LIB
CYCLE NO. 1, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5523.3 SECS.

TAU				
−0.49	0	0	0	2
1.71	0	0	2	−2
0.39	0	1	−2	2
91.57	0	1	0	0
0.23	0	2	0	0
−1.37	1	−1	0	−1
4.12	1	0	0	−2
−3.49	1	0	0	−1
−15.53	1	0	0	0
0.23	1	1	0	−2
0.45	2	−2	0	−2
0.92	2	−1	0	−2
27.67	2	0	−2	0
9.99	2	0	0	−2
−0.45	2	0	0	0
P(1)				
2.86	0	0	1	−2
5533.29	0	0	1	0
−0.22	0	1	−1	0
0.30	0	1	−1	1
−0.40	1	0	−1	−2
123.14	1	0	−1	0
−2.70	1	0	1	−2
1.77	1	0	1	0
P(2)				
−3.16	0	0	1	−2
5511.29	0	0	1	0
−75.13	1	0	−1	0
−1.65	1	0	1	−2
0.51	1	0	1	0
0.28	2	0	−1	0

5051E1 ECKHARDT LIB
CYCLE NO. 2, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5522.3 SECS.

TAU				
−0.49	0	0	0	2
1.66	0	0	2	−2
0.38	0	1	−2	2
91.62	0	1	0	0
0.23	0	2	0	0
−1.37	1	−1	0	−1
−0.42	1	0	−2	0
4.16	1	0	0	−2
−3.49	1	0	0	−1
−16.87	1	0	0	0
0.23	1	1	0	−2
0.45	2	−2	0	−2

5051E1 ECKHARDT LIB
CYCLE NO. 2, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5522.3 SECS.

TAU				
0.96	2	−1	0	−2
14.67	2	0	−2	0
10.00	2	0	0	−2
−0.44	2	0	0	0
P(1)				
2.87	0	0	1	−2
5532.00	0	0	1	0
1.02	0	1	−1	0
0.30	0	1	−1	1
1.24	0	1	1	0
−0.35	1	0	−1	−2
123.17	1	0	−1	0
−2.61	1	0	1	−2
1.57	1	0	1	0
0.24	2	0	−1	−2
1.32	2	0	−1	0
0.31	2	0	1	0
P(2)				
−3.21	0	0	1	−2
5510.60	0	0	1	0
−1.06	0	1	−1	0
1.28	0	1	1	0
−75.67	1	0	−1	0
−1.65	1	0	1	−2
−0.27	2	0	−3	0
−0.22	2	0	−1	−2
0.21	2	0	1	0

5051E1 ECKHARDT LIB
CYCLE NO. 3, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.6 SECS.

TAU				
−0.49	0	0	0	2
1.66	0	0	2	−2
0.38	0	1	−2	2
91.57	0	1	0	0
0.23	0	2	0	0
−1.37	1	−1	0	−1
−0.41	1	0	−2	0
4.16	1	0	0	−2
−3.49	1	0	0	−1
−16.87	1	0	0	0
0.23	1	1	0	−2
0.45	2	−2	0	−2
0.96	2	−1	0	−2
15.32	2	0	−2	0
10.00	2	0	0	−2
−0.46	2	0	0	0

Table 1 (contd)

5051E1 ECKHARDT LIB
CYCLE NO. 3, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.6 SECS.

P(1)				
2.88	0	0	1	−2
5531.54	0	0	1	0
1.03	0	1	−1	0
0.30	0	1	−1	1
1.24	0	1	1	0
−0.35	1	0	−1	−2
122.98	1	0	−1	0
−2.63	1	0	1	−2
1.60	1	0	1	0
0.24	2	0	−1	−2
0.66	2	0	−1	0
P(2)				
−3.19	0	0	1	−2
5509.74	0	0	1	0
−1.06	0	1	−1	0
1.27	0	1	1	0
−75.55	1	0	−1	0
−1.61	1	0	1	−2
0.25	1	0	1	0
−0.23	2	0	−1	−2
0.33	2	0	−1	0
−0.24	2	0	1	0

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0 0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.5 SECS.

TAU				
−0.49	0	0	0	2
1.66	0	0	2	−2
0.38	0	1	−2	2
91.57	0	1	0	0
0.23	0	2	0	0
−1.37	1	−1	0	−1
−0.41	1	0	−2	0
4.16	1	0	0	−2
−3.49	1	0	0	−1
−16.87	1	0	0	0
0.23	1	1	0	−2
0.45	2	−2	0	−2
0.96	2	−1	0	−2
15.32	2	0	−2	0
10.00	2	0	0	−2
−0.45	2	0	0	0
P(1)				
2.88	0	0	1	−2
5531.49	0	0	1	0
1.03	0	1	−1	0
0.30	0	1	−1	1

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.5 SECS.

P(1)				
1.24	0	1	1	0
−0.35	1	0	−1	−2
122.95	1	0	−1	0
−2.63	1	0	1	−2
1.60	1	0	1	0
0.24	2	0	−1	−2
0.65	2	0	−1	0
P(2)				
−3.19	0	0	1	−2
5509.69	0	0	1	0
−1.06	0	1	−1	0
1.28	0	1	1	0
−75.56	1	0	−1	0
−1.61	1	0	1	−2
0.23	1	0	1	0
−0.20	2	0	−3	0
−0.23	2	0	−1	−2
0.32	2	0	−1	0
−0.23	2	0	1	0
TAU				
−0.49	0	0	0	2
1.66	0	0	2	−2
0.38	0	1	−2	2
91.57	0	1	0	0
0.23	0	2	0	0
−1.37	1	−1	0	−1
−0.41	1	0	−2	0
4.16	1	0	0	−2
−3.49	1	0	0	−1
−16.87	1	0	0	0
0.23	1	1	0	−2
0.45	2	−2	0	−2
0.96	2	−1	0	−2
15.32	2	0	−2	0
10.00	2	0	0	−2
−0.45	2	0	0	0
RHO				
−3.05	0	0	2	−2
−10.77	0	0	2	0
0.22	0	1	0	0
23.84	1	0	−2	0
−1.91	1	0	0	−2
−98.36	1	0	0	0
0.50	1	0	2	−2
−0.73	1	0	2	0
−0.37	2	0	0	0
I * SIGMA				
−0.26	0	0	0	2
−2.99	0	0	2	−2
−10.58	0	0	2	0
−23.75	1	0	−2	0

Table 1 (contd)

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300
(OPERATOR POLE = 0.0002126)
I = 5521.5 SECS.

I*SIGMA				
2.45	1	0	0	−2
−100.63	1	0	0	0
0.47	1	0	2	−2
−0.83	1	0	2	0
−0.89	2	0	0	0
U(1)				
204909.91	0	0	0	0
0.29	0	0	0	1
270.06	0	0	0	2
−0.99	0	0	0	3
11.35	0	0	0	4
0.56	0	0	2	−4
−32.24	0	0	2	−2
0.29	0	0	2	−1
692.65	0	0	2	0
−0.31	0	0	2	1
4.37	0	0	2	2
0.29	0	1	−2	−2
−0.70	0	1	−2	0
0.68	0	1	−2	2
1.48	0	1	0	−4
17.76	0	1	0	−2
−0.29	0	1	0	−1
−4.88	0	1	0	0
−7.69	0	1	0	2
−0.26	0	1	0	4
−1.38	0	1	2	−2
0.40	0	1	2	0
0.87	0	2	0	−2
0.54	0	2	0	0
0.61	1	−2	0	0
0.57	1	−2	0	2
−0.29	1	−1	−2	0
−0.53	1	−1	0	−4
0.21	1	−1	0	−3
−7.81	1	−1	0	−2
−0.89	1	−1	0	−1
39.32	1	−1	0	0
10.66	1	−1	0	2
0.31	1	−1	0	4
0.22	1	−1	2	0
6.97	1	0	−2	−2
−44.30	1	0	−2	0
3.21	1	0	−2	2
0.42	1	0	0	−6
27.76	1	0	0	−4
−1.58	1	0	0	−3
−122.74	1	0	0	−2
8.25	1	0	0	−1
−66.86	1	0	0	0
−6.65	1	0	0	1

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300
(OPERATOR POLE = 0.0002126)
I = 5521.5 SECS.

U(1)				
140.07	1	0	0	2
2.13	1	0	0	4
0.61	1	0	2	−4
−10.02	1	0	2	−2
37.97	1	0	2	0
0.56	1	0	2	2
0.49	1	1	−2	−2
0.23	1	1	−2	0
−0.22	1	1	−2	2
3.15	1	1	0	−4
0.44	1	1	0	−2
−42.47	1	1	0	0
1.03	1	1	0	1
−2.49	1	1	0	2
−0.28	1	1	2	−2
0.23	1	2	0	−4
−0.26	1	2	0	0
0.24	2	−2	0	0
−0.34	2	−1	0	−4
−0.51	2	−1	0	−2
9.19	2	−1	0	0
1.29	2	−1	0	2
−0.90	2	0	−2	−2
3.34	2	0	−2	0
−0.36	2	0	−2	2
0.60	2	0	0	−6
24.56	2	0	0	−4
−254.95	2	0	0	−2
1.61	2	0	0	−1
615.36	2	0	0	0
−0.67	2	0	0	1
15.62	2	0	0	2
0.30	2	0	0	4
−1.11	2	0	2	−2
2.31	2	0	2	0
2.24	2	1	0	−4
−9.90	2	1	0	−2
−7.51	2	1	0	0
−0.35	2	1	0	2
−0.33	2	2	0	−2
−0.36	3	−1	0	−2
0.86	3	−1	0	0
−0.82	3	0	−2	0
0.33	3	0	0	−6
0.81	3	0	0	−4
−20.88	3	0	0	−2
41.57	3	0	0	0
1.45	3	0	0	2
−0.70	3	1	0	−2
−0.72	3	1	0	0
−1.68	4	0	0	−2
2.80	4	0	0	0

Table 1 (contd)

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.5 SECS.

U(2)

−124.37	0	0	0	1
2347.59	0	0	0	2
0.33	0	0	0	3
15.42	0	0	0	4
0.24	0	0	2	−3
−49.55	0	0	2	−2
0.63	0	0	2	−1
−697.53	0	0	2	0
0.21	0	0	2	1
−4.53	0	0	2	2
0.30	0	1	−2	−2
−0.66	0	1	−2	0
−3.76	0	1	−2	2
−2.07	0	1	0	−4
−164.26	0	1	0	−2
0.56	0	1	0	−1
−755.42	0	1	0	0
17.92	0	1	0	1
−24.64	0	1	0	2
−0.34	0	1	0	4
0.34	0	1	2	−2
−0.27	0	1	2	0
−8.06	0	2	0	−2
−7.67	0	2	0	0
−0.34	0	3	0	−2
2.51	1	−2	0	−2
3.35	1	−2	0	0
−0.46	1	−1	−2	2
0.71	1	−1	0	−4
−0.27	1	−1	0	−3
28.13	1	−1	0	−2
146.78	1	−1	0	0
15.14	1	−1	0	2
0.37	1	−1	0	4
−0.22	1	−1	2	0
8.27	1	0	−2	−2
42.54	1	0	−2	0
−8.10	1	0	−2	2
−0.52	1	0	0	−6
−39.31	1	0	0	−4
3.15	1	0	0	−3
−4560.34	1	0	0	−2
22.01	1	0	0	−1
22545.04	1	0	0	0
−8.36	1	0	0	1
198.15	1	0	0	2
2.60	1	0	0	4
−3.55	1	0	2	−2
−38.31	1	0	2	0
−0.58	1	0	2	2
0.37	1	1	−2	−2
−4.50	1	1	0	−4

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.5 SECS.

U(2)

0.23	1	1	0	−3
−204.58	1	1	0	−2
−109.29	1	1	0	0
1.27	1	1	0	1
−3.30	1	1	0	2
−0.32	1	2	0	−4
−7.36	1	2	0	−2
−1.13	1	2	0	0
−0.25	1	3	0	−2
−0.20	2	−2	0	−2
0.22	2	−2	0	0
0.39	2	−1	0	−4
−3.91	2	−1	0	−2
−0.37	2	−1	0	−1
10.66	2	−1	0	0
1.52	2	−1	0	2
0.94	2	0	−2	−2
−17.86	2	0	−2	0
−0.43	2	0	−2	2
−0.73	2	0	0	−6
−29.22	2	0	0	−4
1.13	2	0	0	−3
−223.42	2	0	0	−2
2.01	2	0	0	−1
762.54	2	0	0	0
−0.76	2	0	0	1
18.42	2	0	0	2
0.33	2	0	0	4
−0.30	2	0	2	−2
−2.34	2	0	2	0
−2.62	2	1	0	−4
−8.44	2	1	0	−2
−8.80	2	1	0	0
−0.29	2	1	1	2
−0.28	2	2	0	−2
−0.26	3	−1	0	−2
0.95	3	−1	0	0
−0.37	3	0	0	−6
0.27	3	0	0	−4
−20.24	3	0	0	−2
46.99	3	0	0	0
1.63	3	0	0	2
−0.69	3	1	0	−2
−0.79	3	1	0	0
−1.76	4	0	0	−2
3.05	4	0	0	0
0.20	5	0	0	0

U(3)

3.73	0	0	1	−4
−0.33	0	0	1	−3
646.86	0	0	1	−2
−6.59	0	0	1	−1

Table 1 (contd)

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.5 SECS.

U(3)				
−23923.74	0	0	1	0
7.02	0	0	1	1
−152.61	0	0	1	2
−1.56	0	0	1	4
3.88	0	0	3	−2
0.20	0	1	−1	−4
10.43	0	1	−1	−2
13.82	0	1	−1	0
−1.35	0	1	−1	1
12.25	0	1	−1	2
0.42	0	1	1	−4
31.42	0	1	1	−2
15.39	0	1	1	0
−1.04	0	1	1	1
1.70	0	1	1	2
0.51	0	2	−1	−2
1.19	0	2	1	−2
−1.78	1	−1	−1	−2
−7.12	1	−1	−1	0
−1.79	1	−1	−1	2
−1.12	1	−1	1	−2
−9.15	1	−1	1	0
−1.48	1	−1	1	2
−5.18	1	0	−3	0
0.47	1	0	−3	2
3.90	1	0	−1	−4
−0.27	1	0	−1	−3
258.15	1	0	−1	−2
−1423.57	1	0	−1	0
0.59	1	0	−1	1
−33.51	1	0	−1	2
−0.48	1	0	−1	4
6.70	1	0	1	−4
−0.33	1	0	1	−3
231.68	1	0	1	−2
−0.79	1	0	1	−1

5051E1 ECKHARDT LIB
CYCLE NO. 4, ALPHA = 0.0003968
BETA = 0.0006268
GAMMA = 0.0002300

(OPERATOR POLE = 0.0002126)

I = 5521.5 SECS.

U(3)				
−1310.24	1	0	1	0
0.87	1	0	1	1
−19.71	1	0	1	2
−0.28	1	0	1	4
0.76	1	0	3	−2
0.44	1	1	−1	−4
11.58	1	1	−1	−2
5.76	1	1	−1	0
0.82	1	1	−1	2
0.61	1	1	1	−4
10.27	1	1	1	−2
7.26	1	1	1	0
0.32	1	1	1	2
0.41	1	2	−1	−2
0.37	1	2	1	−2
−0.30	2	−1	−1	0
−1.07	2	−1	1	0
0.45	2	0	−3	0
3.11	2	0	−1	−4
−3.10	2	0	−1	−2
−29.96	2	0	−1	0
−2.17	2	0	−1	2
0.68	2	0	1	−4
21.90	2	0	1	−2
−80.13	2	0	1	0
−1.99	2	0	1	2
0.28	2	1	−1	−4
0.34	2	1	−1	0
0.90	2	1	1	−2
0.85	2	1	1	0
0.30	3	0	−1	−2
−1.72	3	0	−1	0
2.06	3	0	1	−2
−5.16	3	0	1	0
−0.34	4	0	1	0

01 EXIT IN PHO2

The semi-literal expansion of the series for H has been developed for the computer using, as input, punched card output for u_i and p_i from LIB. Resulting punched card decks of the H, u_i, and p_i series for different values of the libration parameters β, γ go directly into LIB3 which is a program which calculates everything that PHILB does, but does so without omitting any of the available terms in the series. Moreover, it also calculates the partial derivatives of H (or C) and u_i with respect to β, γ, and two free libration parameters (Ref. 12). (It should be noted that $\partial H/\partial p = \partial C/\partial p$, etc.) The results of this program are now being used, directly in turn, in least squares photogrammetric reductions of lunar photogrammetric plates, and may be used in the reduction of earth-moon laser range data.

Discussion

Van Flandern: Have you attempted any solutions of your own?

Eckhardt: You mean do we have any data?

Van Flandern: Yes.

Eckhardt: I could be long-winded and end by saying no. We have reduced photogrammetric data and solved for the libration parameters, but we get very poor standard deviations, so that the solution really means nothing. We need something more than a moon floating on a plate; we do not have enough constraints. Professor Kopal's group at Manchester has developed a very interesting technique of superposing stars on the moon, which gives a strong constraint on the location of the meridian. The laser is also a promising technique.

Van Flandern: In your proposed plate reductions, you said that the lines pass through the center of mass of the moon. How do you propose to learn where the center of mass is in the reductions?

Eckhardt: The center of mass certainly is not the center of figure. We solve for where the center of mass is. If a small error is made in the location of the center of mass, the effect on that angle will not be too serious.

Eichhorn: Have you introduced any additional parameters beyond those in, say, Koziel's theory?

Eckhardt: Yes, I carry the free libration in longitude.

N69-34639

Optical Tracking of Lunar Spacecraft

H. B. Liemohn
Boeing Scientific Research Laboratories

I. Introduction

Some months ago the *Lunar Orbiter V* spacecraft was oriented to reflect sunlight off its mirrors and solar panels toward earth, and it was photographed over an appreciable fraction of an orbit. The prospect of detecting lunar spacecraft optically presents the possibility of tracking an *orbiter* with respect to the known stellar field and thus determining the lunar center of gravity. Today I want to briefly describe the theory of specular reflection from spacecraft and then discuss its potential application for improving the lunar ephemeris. After my presentation I would appreciate your comments on the value of optical tracking compared with other methods of refining the ephemeris. We are currently evaluating the feasibility and utility of tracking *Apollo* missions and your inputs will be influential.

II. Theory

A plane mirror on a spacecraft acts like a pinhole camera which projects the solar image in a narrow cone of light that is many orders of magnitude brighter than the diffuse reflection from the entire vehicle. Two basic questions must be considered: how large a reflector is needed for detection, and how flat does it have to be? The theoretical details are described in Icarus, Fall, 1968, but I would like to review the results before getting into their application.

The apparent stellar magnitude of the illumination at earth from a mirror near the moon is given approximately by the relation

$$m = +5.7 - 2.5 \log_{10} \{\kappa r a \cos(\alpha/2)\, \sigma^2/(\rho + \sigma + \tau)^2\}$$

Here

κ = the degradation due to atmospheric effects and instrument response

r = the reflectivity of the mirror

a = the area of the mirror in square meters

α = the lunar phase angle

σ = the angular diameter of the sun

ρ = the angular diffraction of a typical surface element

τ = the macroscopic angular deviation of the mirror surface

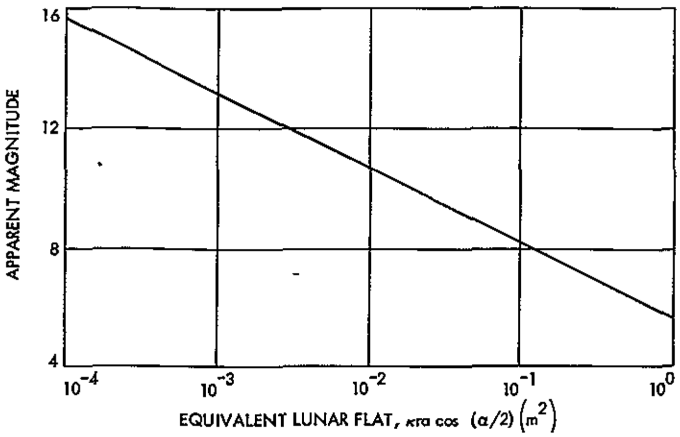

Fig. 1. Brightness of a specular reflector near the moon

The apparent magnitude m is plotted in Fig. 1 as a function of projected area of an equivalent perfect reflector where image cone divergence is negligible ($\rho, \tau << \sigma$). Since large telescopes can detect sources as faint as approximately fifteenth magnitude near the bright moon, we find that reflectors equivalent to only a few square centimeters are discernible.

Because the sun is an extended source, the surface of the reflector does not need to be optically flat, merely specular, i.e., flat to $\lambda/10$ over 100λ ($\rho << \sigma$), where λ is a characteristic wavelength. Such surfaces are readily available with ordinary polished glass or metal. Macroscopic surface roughness and deviations from a flat surface must be taken into account (with τ) as they degrade the intensity.

Surface deviations which diverge the image cone are a mixed blessing. Although they reduce the brightness, the orientation requirements are less stringent with a broad beam. If the deviations are eliminated by proper rigid supports ($\tau << \sigma$), the reflector must be pointed with an accuracy of ± 0.2 deg, or less, to insure detection.

III. Application

The *Lunar Orbiter* V (LO V) was obviously not designed for tracking by specular reflection, and we had to make do with those surfaces that were available. Fortunately the solar panels were approximately coplanar with each other and with the equipment mounting deck where 514 one-inch square mirrors had been placed for thermal control. On the other hand, no special care had been taken to align the individual solar cells or mirrors so that the solar reflection diverged severely. Theoretical characteristics of the LO V reflector illumination are summarized in Table 1.

To photograph the faint reflection from LO V near the very bright lunar disk, Dr. Kuiper and his assistants took very special precautions with their 61-in. reflector. The primary mirror was washed to reduce scatter, a Cassegrain sky baffle was installed in the center of the primary, and a circular mask was centered on the secondary to eliminate multiple reflections. The image was recorded on 52 plates between 12:20:30 and 13:28:50 UT on 21 January 1968 while the spacecraft was approximately 8 to 10 minutes of arc off the bright limb. Its brightness was quite variable with a maximum apparent magnitude of ~12, which was well above the 15th magnitude limit of the instrument. A more complete account of the experiment and photographs of the vehicle and its reflection are presented in Ref. 13. Unfortunately, the star field was very poor so that the position of the spacecraft is very difficult to ascertain. Hence the analysis has not yet been completed. Nevertheless the feasibility and brightness estimates were confirmed.

Table 1. Optical tracking properties

Property	*Lunar Orbiter* V		Apollo	
Reflector	Four solar cell panels	Equipment mounting deck mirrors	Command module window shade	Lunar module exterior panel
Area (a)	4.09 m^2	0.33 m^2	~0.1 m^2	~0.3 m^2
Reflectivity (r)	0.10 (0.50–0.65 μ)	0.93	~0.8	~0.8
Lunar phase (α)	74.6 deg		~70 deg	
Image diameter ($\rho + \sigma + \tau$)	~5 deg	~2.5 deg	~1 deg	~1 deg
Optical response (κ)	0.32 (0.50–0.65 μ)		~0.3	
Illumination (E)	1.4×10^{-11} lux	4.3×10^{-11} lux	6.6×10^{-11} lux	2.0×10^{-10} lux
Apparent magnitude (m)	+11.7		+11.5	+10.3

Currently, I am considering the possibility of tracking *Apollo* missions. The only surfaces on the spacecraft system which appear to be usable as reflectors are the window shades on the command module (CM) and certain flat exterior surfaces on the lunar module (LM). Theoretical estimates of their properties and apparent brightness are also presented in Table 1. Although the reflector areas are not as large as those on LO V, their apparent magnitudes are brighter due to less divergence of the image cone. On the other hand, this narrow image cone requires more accurate orientation of the vehicle.

Because of the complex program of the *Apollo* missions, it would probably be difficult to arrange for orientation of a manned CM or LM for the required time interval. The CM tracking might be achieved with ground control during a rest period for the astronauts. A more logical opportunity would be to track the abandoned LM after the mission.

The low-altitude equatorial parking orbit frequently described for *Apollo* missions is not amenable to optical tracking. Fortunately, eccentric orbits are being planned for the early missions which would be sufficiently far from the bright disk to permit optical detection. In later missions, the abandoned LM will most certainly have extra fuel available so that it could be boosted into a useful orbit and oriented at the convenience of the ground observers.

Why bother to track optically? Certainly radar data are much more accurate. However, the optical data are in the plane perpendicular to the line of sight which actually complements the range and range-rate data. By repeatedly determining the position of the spacecraft with respect to the star field at several locations along the orbit, the location of the lunar center of gravity might be determined more accurately. The current lunar ephemeris, which is based on combined astronomical and radar data, has an uncertainty corresponding to a standard deviation of approximately 200 m. With optical tracking, the instantaneous right ascension and declination of a lunar spacecraft can be determined to approximately 0″.1, which corresponds to approximately 200 m. By gathering data from several orbits and applying statistical methods, the position of the center of gravity in the perpendicular plane could be more accurately determined by up to an order of magnitude.

Therefore, it appears that optical tracking may provide useful addition to lunar ephemeris refinements.

Discussion

Eichhorn: That 0″1 will take some doing.

Liemohn: This is the estimate that I get from Miss Roemer at Tucson.

Kopal: This estimate depends very much on the quality of the optics employed for tracking. If you have to use a reflector such as the Catalina instrument, this gives a poor field, and you will have to take into account at least quadratic terms in the plate constants. This means that you need a minimum of 12 stars.

Eichhorn: There is also a problem with reference stars, which will have to be taken out of the Astrographic Catalogue. The Astrographic Catalogue is, on the average, about 70 years old. The influence of proper motion on these stars is, on the average approximately 1″. The plate constants are presently typically such that there is a systematic error of approximately 0″.5 left in every position that you calculate. The field that you will get is quite small and, before you get 0″.1 out of this, there is a lot of work that must be done in Astrographic Catalogue astrometry. You can get a *relative* position of 0″.1, but this does *not* mean that you can get 0″.1 with respect to any well-defined system presently in use.

Sconzo: This is the biggest handicap in using this catalog.

Liemohn: This sounds like a very serious obstacle that I was not aware of.

Kopal: There may be another obstacle. When this experiment was conducted in January, the spacecraft had to be re-oriented several times to make it reflect light specularly. Did this operation involve perturbing the center of gravity of the spacecraft?

Liemohn: Once the spacecraft has been oriented, it should stay properly positioned for at least most of an orbit.

Eichhorn: For the past six years, I have been trying to interest people in a new reduction and new photography of the Astrographic Catalogue. At a rough estimate, it could be done for approximately $1,000,000. The data that we have on stars whose positions are well known with respect to a well-defined fundamental system, namely FK 4, reach only to approximately the ninth magnitude. In the past, the suggestion for better astrographic positions has met with the arguments: "nobody will ever need them," or "we do not need it now, therefore, we cannot justify it." It seems that the situation has arisen now where it would be very advantageous if these positions were available. Einar Herzsprung once said that "we do not *know* what the next generation of astronomers will want, but we know that they will want it much more accurately."

Discussion (contd)

Herrick: I think the speaker is waging a battle that we should all support.

Kopal: This matter could soon become much more pressing. In the years to come, we may see lunar satellites launched for the special purpose of permitting us to track the moon much more accurately than we can track the moon directly. A satellite anchored gravitationally to the moon and having stellar appearance can be used to determine the center of mass of the moon much more accurately than by any other means. However, to accomplish this, you need a sufficient number of absolute reference points.

Warner: I will disagree with anyone who says that it is only the new applications that require better catalogs. On my desk are four plates of Icarus taken in June. The field is approximately $\frac{1}{4}° \times \frac{1}{4}°$, and in each case there is one, and *only* one reference star that is in the SAO Catalog.

N69-34640

Laser Ranging to Optical Retro-reflectors on the Moon

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University of Maryland
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Abstract[1]

The accuracy with which one can determine the various parameters entering into the lunar orbit theory, the librations, and the location of the reflector with respect to the center of mass is discussed (see Refs. 14 and 15).

Discussion

Mulholland: What do you regard as adequate knowledge of the masses of the major planets?

Bender: It seems to me to be very high. You mentioned during lunch that the present uncertainties are such that the uncertainties in the orbit will come down to perhaps 100 m. I am quoting you very loosely. Here, we are talking about much higher accuracies; therefore, it seems difficult to believe that the present accuracies of the planetary masses will be high enough to make these corrections.

Mulholland: I think I have been misquoted. I said that I did not believe that the errors in the extant theory would be significantly reduced if it were corrected only to the extent of introducing the true masses of the major planets.

[1]Abstract only (no manuscript available).

Bender: Well, that is a quite different statement. It sets an upper limit on the degree of uncertainty presently due to the masses. We have not analyzed this ourselves.

Mulholland: I might also point out that the masses now included in the lunar theory are very far from the most adequate values now available. I think that we can expect that it will not be too long before there exists a lunar ephemeris with more adequate masses.

Bender: Quite possibly a numerical integration?

Mulholland: Quite possibly.

Deprit: When you speak of the planetary perturbations, do you mean the direct or indirect?

Bender: To be precise, what I mean is everything left out of the Hill-Brown theory.

Discussion (contd)

Deprit: The moon acts as an amplifier of planetary perturbations on the earth. Perhaps more than improved planetary masses, you need a better *orbit* for earth.

Bender: Presumably, the radar astronomers will offer some help with that problem.

Kopal: Would it be possible to use your observations to improve masses of the major planets?

Bender: I really have no idea.

Mulholland: The improvement in the masses of the major planets that could be obtained in this way is insignificant compared with that which can be obtained from the analysis of planetary radar data.

Unidentified: I did not get the status of the retro-reflectors. Are they being built now?

Alley: NASA approval was given well over a year ago to get hardware for the experiment. As yet they have not signed a hardware contract. We feel that it can be ready in time for the first *Apollo* lunar landing, which may occur a year from now. The program is formally approved; it is just a matter of managerial inactivity.

Sconzo: The best way to improve the masses of the planets is through better theories of the natural satellites.

O'Handley: Probably for the masses of the inner planets, one cannot improve beyond the values given by spacecraft, at least not through any astronomical means. For Jupiter, it is better to use asteroids; for Pluto, one must use Neptune. I think numerical integrations with variational equations for the mass partials are giving us far more concrete results than we ever expected from the study of any particular satellite.

Alley: I would like to make one last remark. We feel that this is a natural experiment for significant international cooperation. We have had some overtures from France, the U.S.S.R., and other countries. I would like to re-emphasize that we welcome such interaction.

Eckhardt: AFCRL is involved in this program, too. We expect to have our laser operating this winter, but we are severely limited in what we can do without the cube-corner reflectors. What we intend to do is to range directly off the sub-radar point on the moon.

N69-34641

New Corrections to the Lunar Ephemeris

Thomas C. Van Flandern
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A year ago, preliminary corrections to the moon's orbital elements and the constants of the lunar theory were presented at a seminar sponsored by JPL (Ref. 16). These corrections, based on approximately 700 meridian circle observations from 1956 through 1966 (Ref. 17) and a few grazing occultations, were determined using classical methods for forming the conditional equations. The observations were referred to the moon's center of mass by the use of Watts' limb corrections (Ref. 18), while ΔT was determined with respect to the A.1 Atomic Time Scale. In addition, the observations were referred to a fundamental coordinate system closely related to that of the FK4 (Ref. 19).

This work was part of a major project at the U. S. Naval Observatory involving the reduction of approximately 20,000 lunar occultations from 1950 up to the present. The reduction process is going rather slowly, partly because the raw observational data are so scattered, but primarily because the limb corrections for each point of contact must be applied manually. We hope that this process can be mechanized within the next year, thus speeding the reduction process. Although we have extended the number of reduced observations by only approximately 1,000 in the past year, quite a bit has been done in the area of analysis and interpretation of the data, and modeling of the residuals. This has led to a significant revision of the corrections.

The most important change has been the departure from the classical method of forming conditional equations. Analytical partial derivatives have been obtained by analytical differentiation of the literal expressions of the Brown Lunar Theory. Enough terms were retained in each of the partial derivative series to give all corrections with sufficient accuracy that the corrected theory would be valid to 0.″01. The usual procedures were followed for obtaining a set of solution parameters of better determinateness than the orbital elements.

Further, the expression for ΔT was altered from $30^s.71 + \text{A.1} - \text{UT2}$ to $32^s.15 + \text{A.1} - \text{UT2}$, to bring it into conformity with current usage. This has introduced a large compensating correction to the mean longitude of the moon. These two changes arise from the longitude system of the lunar theory whose origin is approximately Newcomb's equinox rather than that of the FK4.

The change in procedure has permitted much better modeling of the lunar motion. We have also taken into account a good many more parameters. Largely this, rather than the small increase in the number of observations available, has led to considerable improvement in the values of the parameters. For comparison, the preliminary values given last year are listed here along with the new values (Table 1). The time T is measured in centuries from 1958.0.

Table 1. Corrections to parameters for the epoch 1958.0

Parameter	Symbols	New value	Preliminary
Inclination	ΔI	$-0''.29 \pm 0.03$	$-0''.33 \pm 0.1$
Eccentricity	Δe	$+0.03 \pm 0.02$	$+0.03 \pm 0.02$
Argument of perigee	Δw		$-0.8 \pm 1.$
Longitude of perigee	$\Delta \tilde{\omega}$	-1.8 ± 0.2	
Obliquity	$\Delta \epsilon$	-0.13 ± 0.03	-0.18 ± 0.1
Right ascension system	$\Delta \alpha_0$	-0.24 ± 0.03	-0.36 ± 0.2
Longitude of node	$\Delta \Omega$	$+2.55 \pm 0.3$	$+4. \pm 1.$
Mean longitude	ΔL	-0.816 ± 0.017 $+(0.732 \pm 0.352)\,T$	
Equator	$\Delta \delta_0$	$+0.28 \pm 0.08$	
Coefficient of sin L in longitude	ΔX	-0.14 ± 0.03	
Latitude system	$\Delta \beta_0$	-0.31 ± 0.02	
Solar mean anomaly	$\Delta l'$	$-15.5 \pm 9.$	
Solar eccentricity	$\Delta e'$	-0.25 ± 0.15	

The values $\Delta\tilde{\omega}$ and $\Delta\Omega$ have been referred to what I will call an "ideal" equinox, the equinox of this solution, which is independent of the FK4 equinox. For the same "ideal" coordinate system, $\Delta L = -1''.036 \pm 0''.035 + (0''.732 \pm 0.352)\,T$ and $\Delta\beta_0 = -0''.05 \pm 0''.08$.

The newly considered unknowns form an essential part of the solution. The ΔL correction has no meaning unless we talk about the time scale used in the solution. Basically, it is treated as an Ephemeris Time (ET) scale, but in reality it is an Atomic Time (AT) scale. This AT scale gives a good approximation to the currently accepted ET scale; therefore, I adopted it and solved for the corresponding correction to the mean longitude of the moon. It would have been just as well, in principle, to regard the mean longitude as correct and solve for the constant and rate of the difference between ET and AT. That was, in fact, done also, and it was determined that the ET scale and the AT scale, as derived from the moon, differ by approximately $1^s.3 \pm 0^s.6$ per century. This is well within the uncertainty of the length of the ephemeris century, which has to be derived from observations of the sun. The value adopted for the empirical secular acceleration of the moon, which is due to tidal friction and other uncertainties, will affect the length of the ephemeris second as derived from the moon. Thus, it seems sensible to use an AT scale (since there is no observational evidence for a true difference in the two scales), and to correct the mean longitude accordingly. Nonetheless, the size of this correction may be surprising. The problem has to do with the equinoxes, the origin of longitudes used in the lunar ephemeris and the origin of the system of the observations themselves. Previous reference was made to this (Ref. 16) and, while the conclusion was correct, the explanation was reversed. The longitude system of the Improved Lunar Ephemeris, of the lunar data in the American Ephemeris and Nautical Almanac, and of the Eckert solution is based upon the equinox of Newcomb which differs by approximately $0''.8$ from the FK4 equinox upon which the observations are based. One can either correct ΔT or the mean longitude, but it is incorrect to take lunar positions straight out of the ephemeris and compare them directly with observations made on the FK4 system without correcting one or the other. We have chosen to correct the mean longitude and this also affects those quantities that are longitude-dependent.

The correction to the equator $\Delta\delta_0$ is the excess of the distance of all points completely around the equator over or under 90° (not a tilt, but a compression of the equator). I shall return to this in a moment. The correction ΔX is a companion to the corrections to the equator, equinox and obliquity.

It is not very practical to determine corrections to the elements of the earth's heliocentric orbit from observations of the moon, and I do not really propose to do it. It is much more accurate to use observations of the sun, and better results have already been obtained by such analyses. Nonetheless, the corrections have not yet been carried over into the lunar theory, and the corrections are so large that their effects can be seen in the moon. There has not been a complete accounting for the effects of the transition from UT to ET in 1960 on the constants of the lunar theory and the theory of the earth. They require a new fitting of these theories to observations.

The corrections that amount to corrections to the coordinate system of the stars ($\Delta\alpha_0$, $\Delta\delta_0$, ΔX) all are larger than had been expected. They represent distortions of, or orientation errors in, the FK4 system. There is considerable question about accepting them at face value. All of them could be affected systematically by limb corrections if Watts' charts are systematically biased. They might be affected by the star positions used, which cannot all be FK4 stars (in fact, only a small percentage of them are). There may be systematic instrument errors in the meridian circle observations. This problem is still being considered. Other investigators have derived similar corrections from observations of the moon (Refs. 19 and 20).

At the moment, this is the only object that gives so large a correction to the equator, but, in theory, the moon should be able to give a more accurate correction to those quantities than could the observations of the other ordinary bodies in the solar system. Whether we are for the first time seeing something that is significant, or whether there remain systematic errors in the solution, remains to be seen.

The primary area in which the present work will be concentrated in the coming year will be a re-modelling of the star coordinate system corrections in the solution. It is just possible, for example, that we are seeing some residual star-streaming effect and that these corrections to the coordinate system should be re-discussed in terms of new parameters to represent some new kinds of distortion not previously recognized.

Discussion

Eichhorn: I thought the FK4 positions were derived without regard to any hypothesis regarding the systematic behavior of the proper motions.

Van Flandern: Are they not dependent on an adopted value of the constant of precession?

Eichhorn: Yes.

Van Flandern: Are they not dependent on certain assumptions concerning the location of the pole, the drift of the geophysical pole?

Eichhorn: No.

Van Flandern: Indirectly they are, because the latitude of the observing station will change with that drift. Even though this is usually taken into account for solutions from any one observing station, there are problems in tying in the latitudes of the different stations with one another. If there is continental drift, if there is systematic secular motion of the earth's pole, it may possibly have biased some, or all, of the observations. I do not advance this as a working hypothesis, just as a plausible one.

Eichhorn: If you combine $\Delta\delta_0$ with ΔX, you get a correction to the equator that exceeds $0''.4$?

Van Flandern: That is correct.

Eichhorn: Could it be possible that this $\Delta\delta_0$ is a discrepancy between the geometric center of the moon and this measure of the center of gravity of the moon that is described by the orbit?

Van Flandern: Supposedly, that discrepancy has been absorbed into the Watts limb corrections. That is, the spherical datum to which the limb corrections refer has a geometric rather than gravitational center, to be sure, but it is a point fixed in the moon.

Eichhorn: But still, that point need not be identical with the center of gravity.

Van Flandern: True, but supposedly this $\Delta\beta_0$ should represent the size of this discrepancy.

Deprit: When you mention the theory of the moon, you mean the solar part?

Van Flandern: Yes.

Deprit: And you have no idea how the planetary perturbations will affect the solution?

Van Flandern: In principle, it is true that we have no idea, but in practice we expect that the solution parameters will have periods different from the planetary perturbations.

Sconzo: I do not wish to make a bad prophecy, but I think that, when you analyze all of the occultation data, you will find an entirely different set of values.

Van Flandern: I think not. We have already shown that the origin of these corrections can be almost entirely accounted for in terms of the transition from UT to ET and the introduction of Watts' limb corrections, both of which model out serious systematic errors that were present in the older investigations.

Lieske: Are you ascribing your correction to the mean longitude to a correction to the determined value of ΔT or to an actual correction to the rate of ET?

Van Flandern: Both are represented. The constant is equivalent to a correction to the constant to be added to AT; a correction to the mean motion of the moon is equivalent to a rate difference between the ephemeris and atomic second. But when I say "ephemeris second" in that context, I mean the unit of time applicable to the lunar ephemeris, which is *not* the ephemeris second as defined.

Lieske: Then you are not correcting the defined ephemeris second, you are just adjusting your moon to match the tropical year as defined.

Van Flandern: I cannot say anything quite that sophisticated. The definition of AT was done in terms of the moon, which presumably was calibrated in terms of the sun. This is a correction to the moon to bring it back into conformity with AT. We have yet to re-reduce observations of the sun to find out what ET is really doing. Right now, we are just bringing the moon into an arbitrary, but consistent, system based on an arbitrary definition of the epoch of AT and the length of the atomic second.

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N69-34642

Numerical Integration of the Lunar Motion

J. Derral Mulholland
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I find myself rather in the position of the fabled military commander who is always perfectly well-equipped, both in hardware and strategy, to fight the previous war. I believe I said last year that now that we have a (what was then) new lunar ephemeris, we were beginning to get some idea how good the previous one had been (Ref. 22). Essentially what I want to talk about today is the process by which we are beginning to get some idea of how good the ephemeris that we introduced last year is. Unfortunately, we cannot be as optimistic about it as I was last year. Just to summarize, that ephemeris was JPL Lunar Ephemeris No. 4 (LE 4), which is currently incorporated into the present Export Ephemeris DE 19 (Refs. 23 and 24). It consisted of the basic Improved Lunar Ephemeris (ILE) (Ref. 25), modified by several corrections. Most prominent among these were the transformation corrections of Eckert, Walker, and Eckert (Ref. 26), but other corrections accounted for 1964 changes in the IAU System of Astronomical Constants, as well as a couple of other comparatively minor corrections.

At that same meeting, some other JPL people talked about the behavior of the spacecraft residuals obtained with the "old" ephemeris LE 2 and with LE 4. The results with LE 4 were not as good as we might have desired them to be. There were still large residuals—on the order of several hundred meters in the range to *Lunar Orbiter* spacecraft—which could not be resolved with that ephemeris. Also, the solutions that had been made to determine the locations of *Surveyor* spacecraft on the surface were rather inconsistent; different ways of obtaining the solutions gave results that differed by several kilometers. The "best" solution, if I recall correctly, placed the spacecraft about 6 km *below* the surface. For some reason, this is unacceptable to the spacecraft people.

On the other hand, Clemence had noted that the lunar theory had never been carried to the same stage of completion in the planetary perturbations as had the main problem. This was even true of Brown's work, and the work of Eckert and his collaborators has widened the disparity by improving the treatment of the main problem. Clemence made what appears now to have been a remarkably good estimate of the magnitude to be expected in the residuals from this cause.

Largely as a result of these discussions, several efforts were undertaken to integrate the lunar motion numerically and to fit the integration to some suitable data. Because of the existence of operational software and because of the problems associated with using the lunar observations, we decided that the most reasonable course for us to follow in the beginning was to fit the integration to the lunar theory, as represented by the JPL ephemeris. If we maintain consistency between the model of the theory and that of the integration, then this process is equivalent to the determination of gravitational defects in the lunar theory. Figure 1 shows the results of the first attempts that were made at fitting an integration to the lunar theory (Ref. 27). The integration covered only a two-year interval, which was chosen because it encompassed all of the lunar mission activity to date. One is struck immediately by the nature of these residuals. There is quite an obvious modulation of the amplitudes, and it is not difficult to convince oneself of the presence of planetary synodic periods. Also, there are short-term variations that have periods of approximately 10 days[1].

[1]This feature was discovered nearly simultaneously by Devine and Sturms (Refs. 28 and 29).

Periods near ten days can arise in the lunar motion through an entire class of periodic terms, numerous members of which appear in the ILE. It seems evident that the residuals represent the cumulative effects of many more such terms that have been omitted in the theory. What made these findings more interesting was the knowledge that the *Surveyor* residuals exhibited a ten-day periodic nature, which could not be understood when they were discussed last year. As a consequence, an ephemeris incorporating the integrated lunar data was made available for use by analysts using the *Surveyor* data. Figure 2 shows the first results (Ref. 30) of the use of JPL Lunar Ephemeris No. 5 (LE 5). Admittedly, very systematic trends remain in the data, but the roughly ten-day trend has been eliminated. It seems evident that the spacecraft tracking data are in fact confirming and monitoring gravitational defects in the theory. Figure 3 compares the range residuals obtained from a reduction of

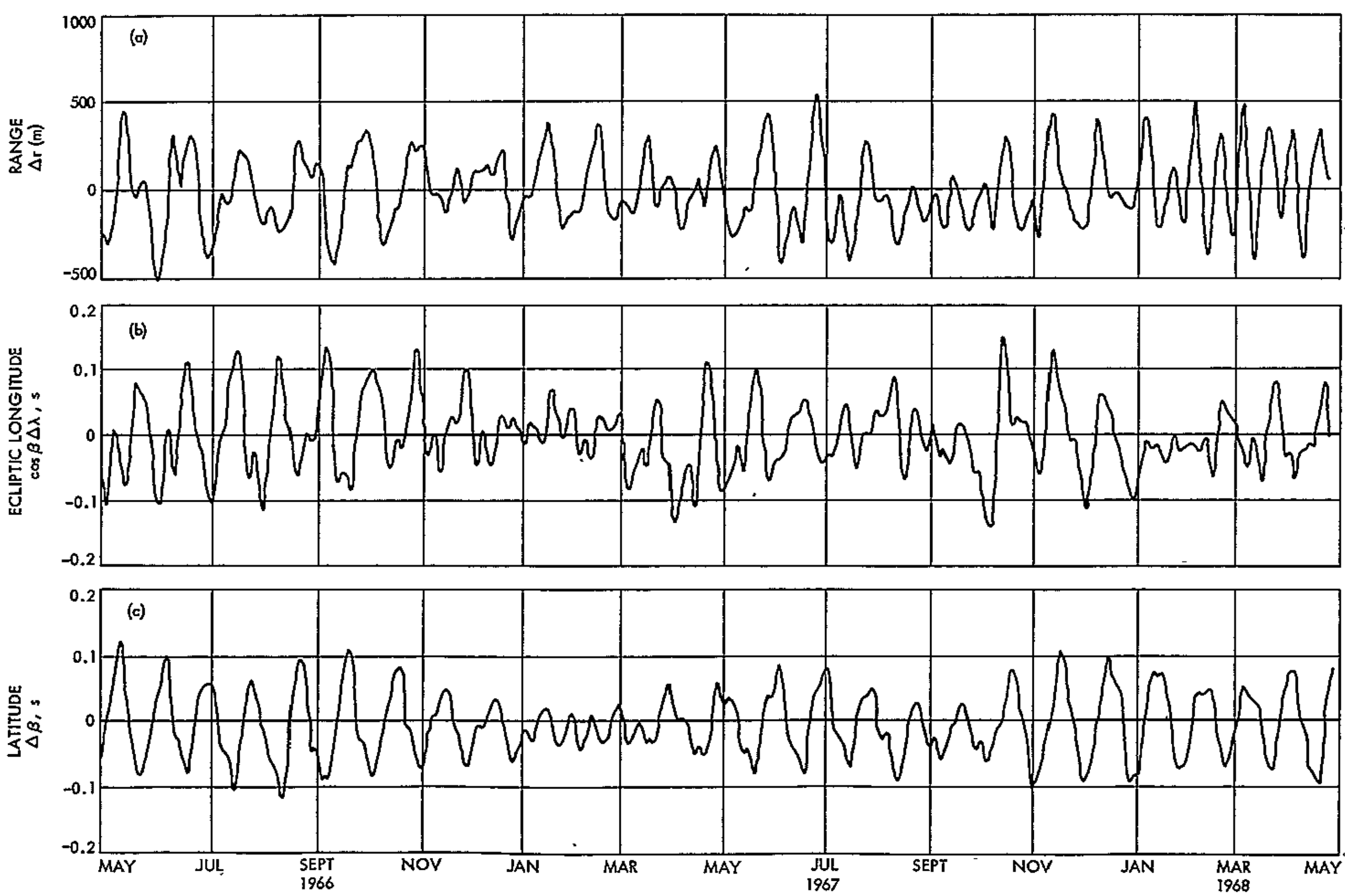

Fig. 1. Differences (LE 4–LE 5) between the current JPL lunar ephemeris and the numerical integration, in geocentric ecliptic spherical coordinates

Lunar Orbiter data (Ref. 30) using LE 4 with the range differences between LE 5 and LE 4. Thus, two different data types from two different spacecraft verify beyond question that the defect does exist in the theory and that, at least over this two-year time span, it does look something like the residuals of Fig. 1.

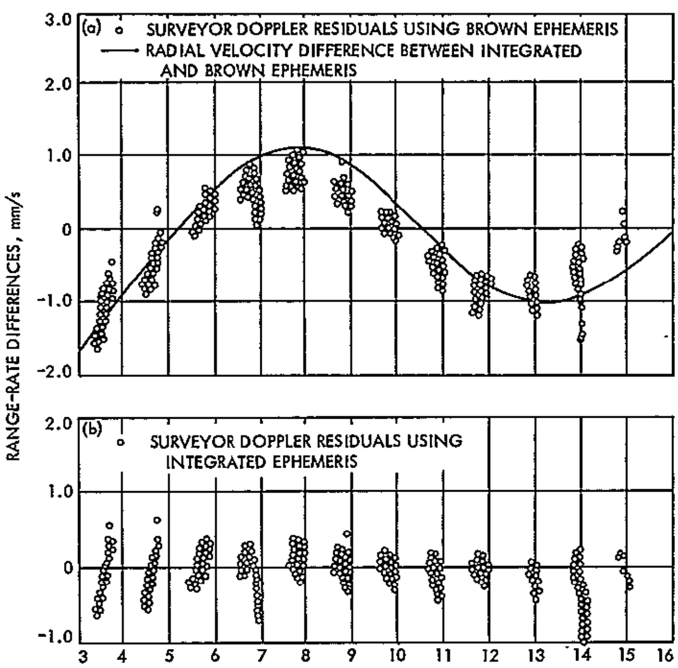

Fig. 2. Doppler residuals from *Surveyor I* on the lunar surface

Now, it seemed, all that we had to do was to run the integration over a longer time span and fit iteratively to the ephemeris until the process converged. A longer interval of fit was necessary because the long-term characteristics of the motion could not be properly modeled otherwise. The nodal period (18.6 years) was regarded as an absolute minimum; the interval adopted was JD 243 3280.5 − 244 0800.5, slightly more than 20 years.

When we undertook the extended integration, we merely reaffirmed the proposition that, when one attempts to explain a phenomenon with an extremely defective model, one gets extremely defective results.

The moon is a highly perturbed object, and the standard differential correction procedures are based on formulae that assume Keplerian motion. In most "ordinary" problems of astronomy, one can get away with this—the planets, for example, are remarkably well-behaved, as are many of the other objects in the solar system. But when dealing with an object that is highly perturbed, one must use perturbed partial derivatives. There are three ways of obtaining these derivatives. Mr. Van Flandern has already mentioned his derivation of analytic expressions for the partial derivatives from the literal theory. One can perform a series of integrations, systematically varying the starting conditions, and difference the results to produce the finite difference quotient approximations to the derivatives. Most rigorously, one can integrate the differential equations that describe the partial derivatives of the equations of motion, frequently called the variational equations. Each of these three procedures have

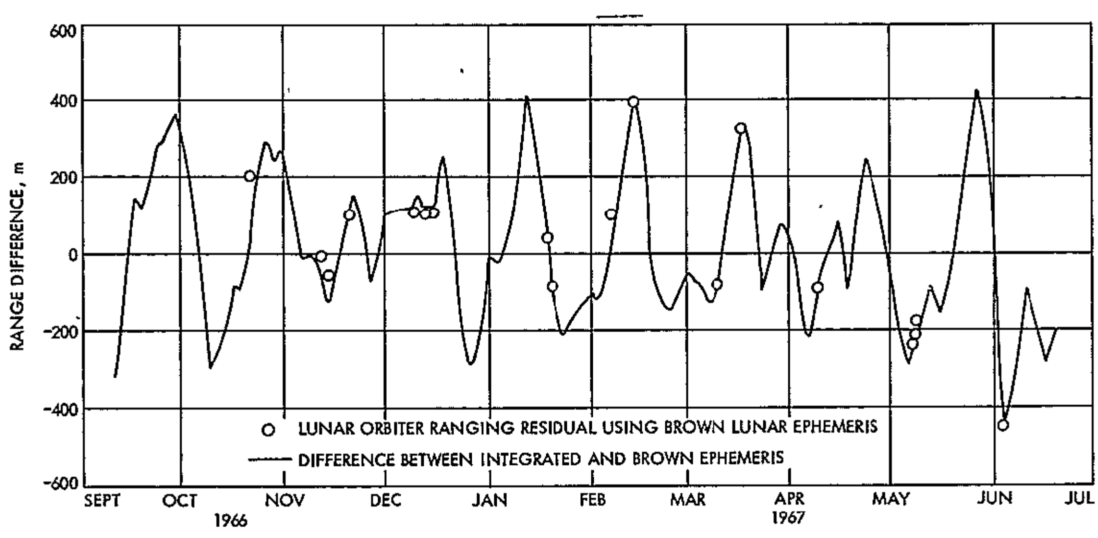

Fig. 3. Range residuals on the geocentric radial coordinate of the moon, 1966–1967

advantages and disadvantages. Again, influenced by the availability of computer software which could be easily modified to our purposes, we chose the finite difference approach as the cheapest for the current work.

Figures 4 through 7 show successive iterations in the process of fitting integrations to the theory with finite difference partials. The starting conditions for the 5-year integration in Fig. 4 were obtained from a converged series of 2-year fits. This is evidenced by the way in which the longitude residuals behave; they are quite good over the previous fit interval and then begin diverging. The general behavior of the latitude residuals is induced by the longitude drift. A differential correction was made

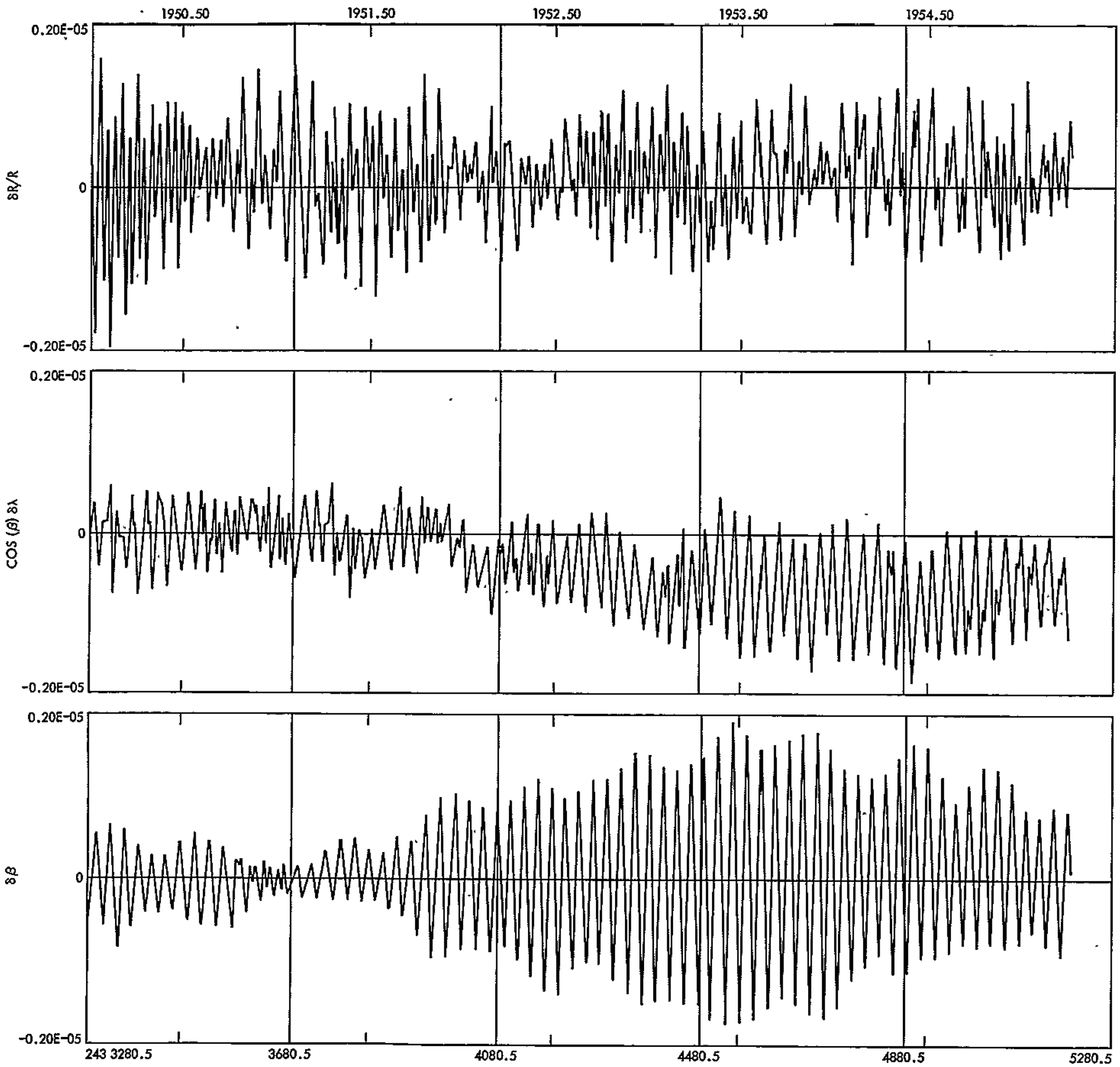

Fig. 4. Residuals of 5-year integration fit to LE 6, iteration 1

and the resulting starting conditions were used to perform the 5-year numerical integration whose residuals are shown in Fig. 5. The effectiveness of the correction may be judged by the longitude drift, which is three orders of magnitude smaller than in the previous integration. At this point, we began feeling better about our differential correction process. The corrections based on the residuals from this integration were rather small.

Figure 6 represents the first attempt at a 10-year integration. Again, we see that the integration fits very nicely over the interval of the previous fit, with some subsequent divergence in the longitude. This integration was terminated at approximately 9 years by a tolerance limit on the sum of squared residuals. After a differential correction, the longitude residuals (Fig. 7) from the second 10-year integration exhibit a long-period behavior with

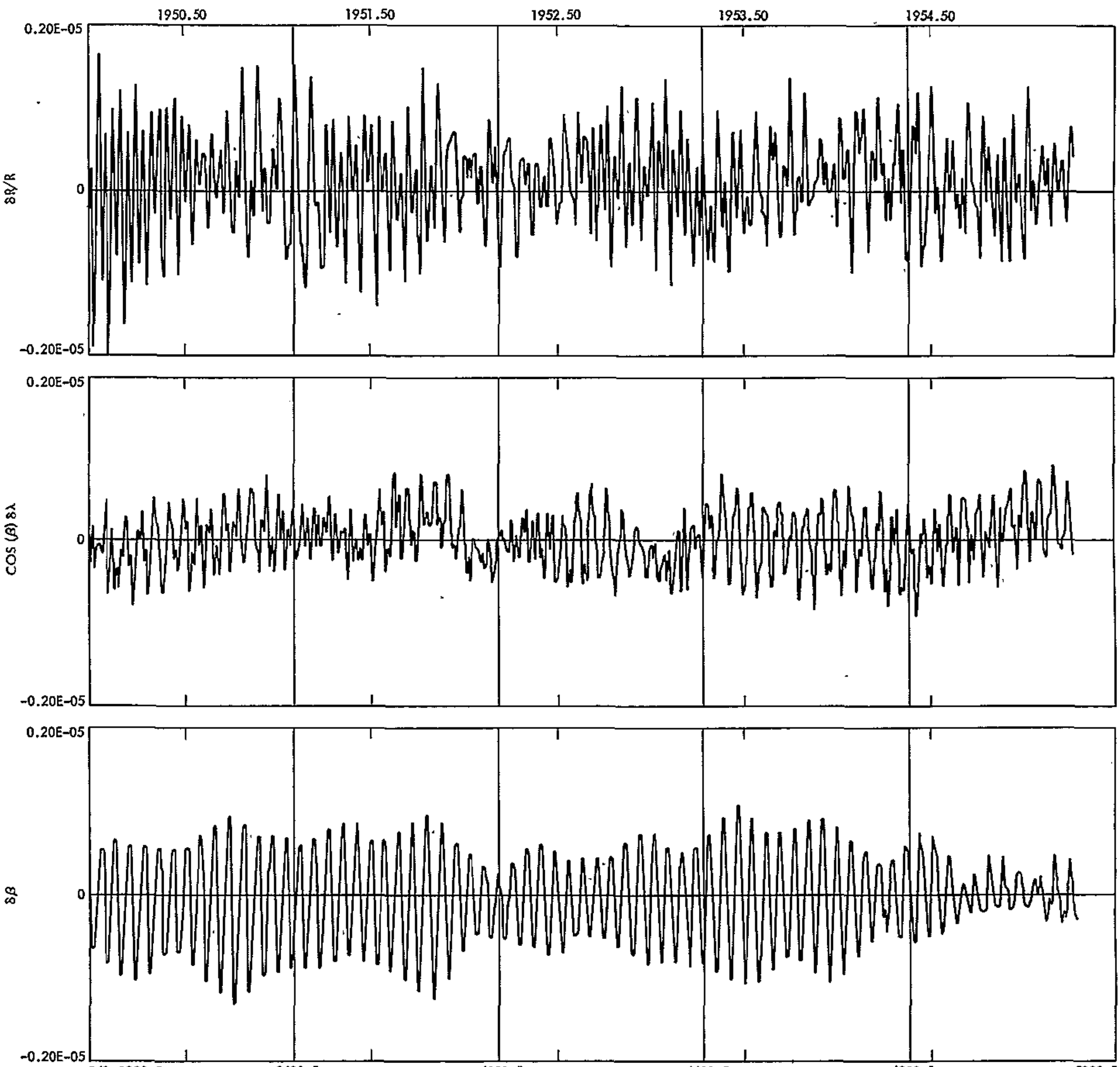

Fig. 5. Residuals of 5-year integration fit to LE 6, iteration 2

approximately a 9-year period. The latitude residuals show what looks like some kind of beat phenomenon, but I do not think it is, for the following reason.

These residuals represent attempts to fit the lunar theory as it exists on our ephemeris tapes. Unfortunately, this is not a completely gravitational theory, because the motion of the moon is not completely gravitational. There is a component in that motion that is caused by tidal friction acting in the earth. This causes the earth to slow down, losing angular momentum, which is transferred to the moon. The moon gains angular momentum and, paradoxically, slows down. This effect should be removed from the ephemeris before the integration is fit to it, because the effect cannot be modeled properly. In the longitude residuals of Fig. 7, this effect will be represented

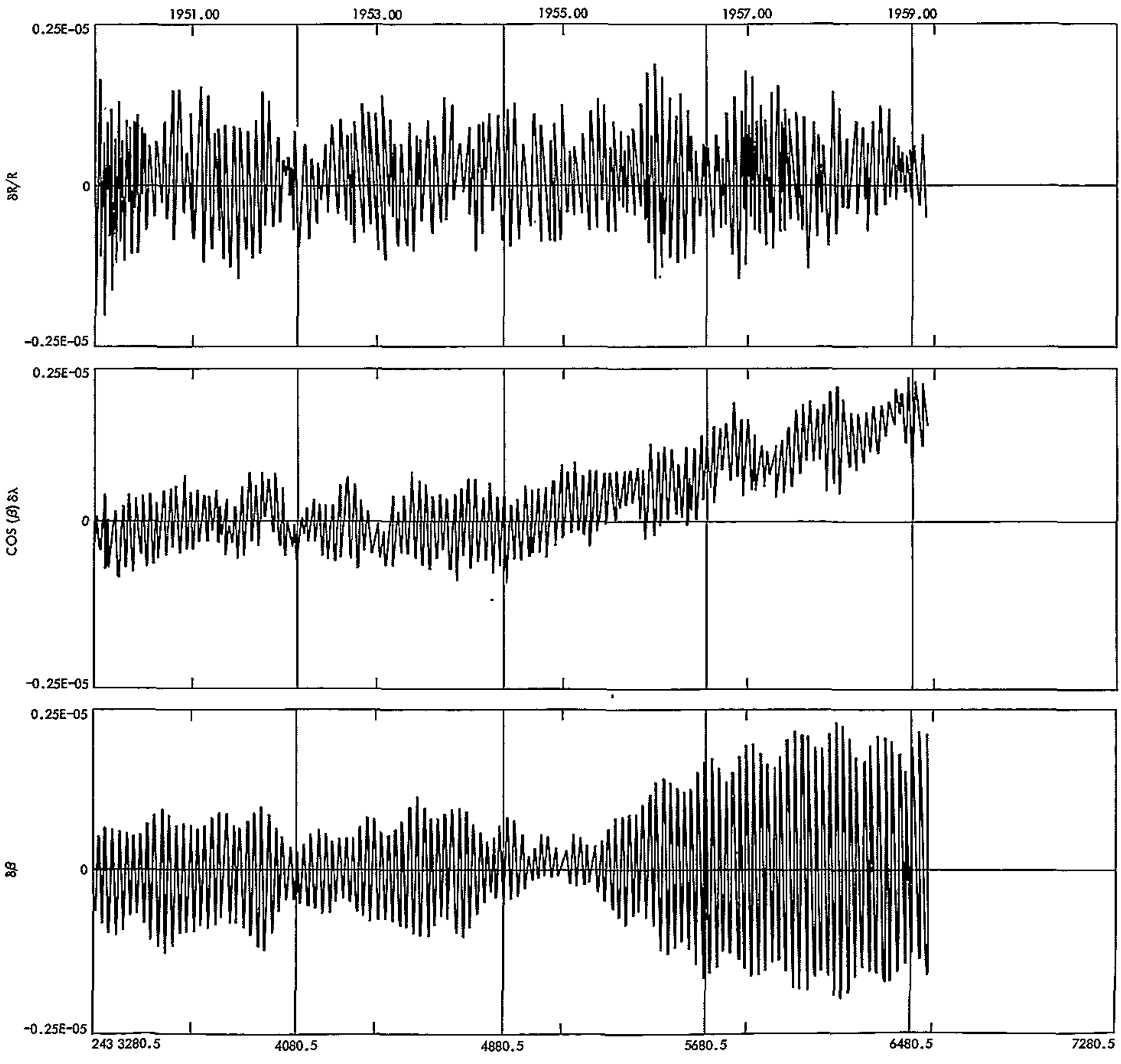

Fig. 6. Residuals of 10-year integration fit to LE 6, iteration 1

by a parabolic arc due to the tidal friction term. I think that the nodes in the latitude residuals are evidences of zero crossings of this parabolic component in the longitude residuals. That conjecture seems to have been verified by subsequent work, but too recently for me to have residual plots available. An ephemeris has been prepared from which the tidal acceleration terms have been removed. A differential correction to this ephemeris produced residuals which did not seem to show the 9-year period in the longitude and the envelope of the latitude residuals was very nearly flat.

We feel that we are well on the way to having a satisfactory fit over 20 years of an integration to a gravitational ephemeris. However, once this is achieved, it will

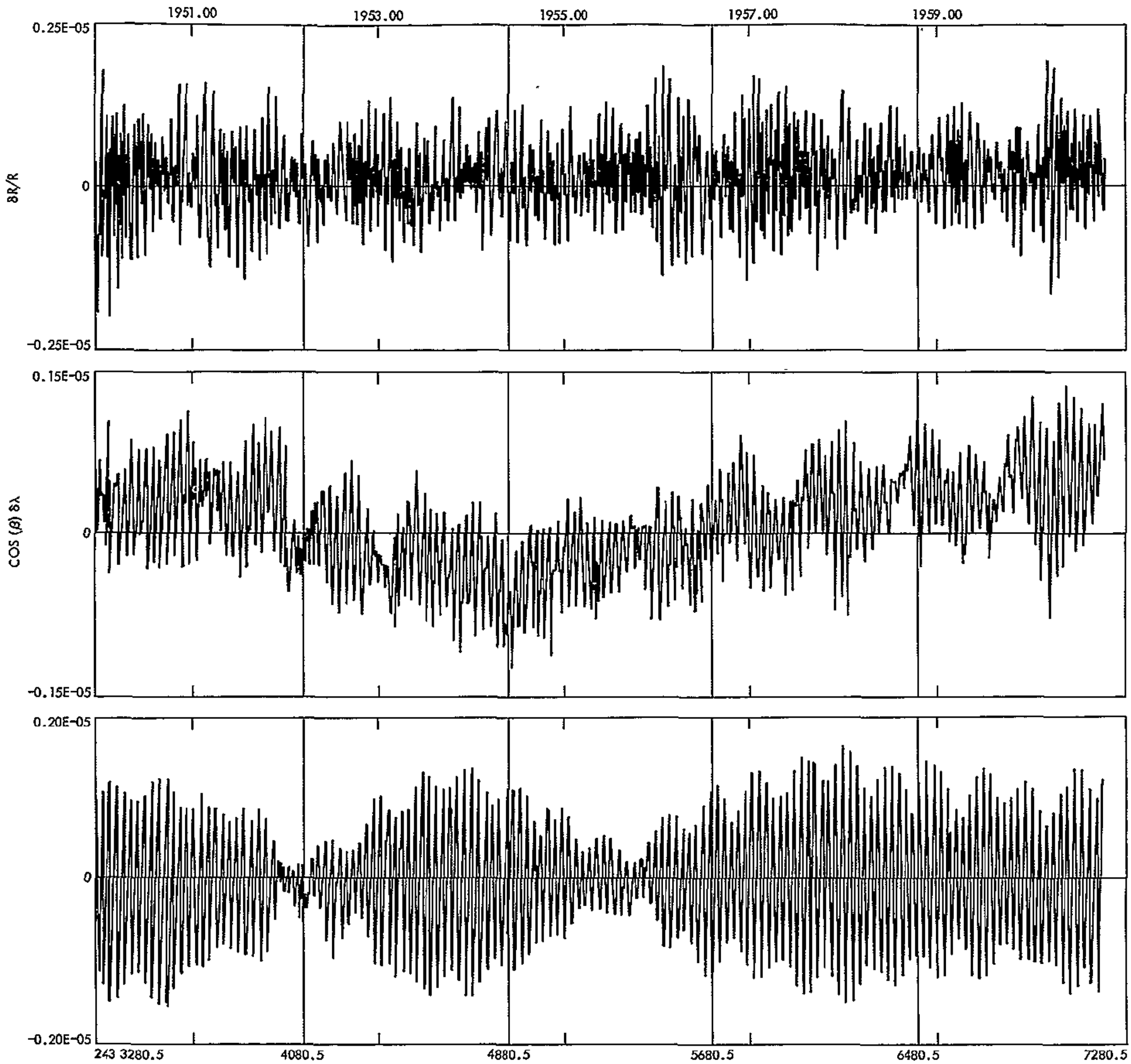

Fig. 7. Residuals of 10-year integration fit to LE 6, iteration 2

be necessary to restore the non-gravitational part algebraically.

There are some other problems associated with the ephemeris that I would like to mention.

Over the past few years, it has been noted that a consistency problem exists in the longitudes of tracking stations as determined from the tracking of various spacecraft. The solutions, based on several planetary spacecraft, are more-or-less compatible with one another. Those based on several lunar spacecraft are more-or-less compatible with one another, but the two sets are incompatible by something on the order of 20 to 25 m. Just out of curiosity, I recently applied to LE 5 an approximation to the ΔL due to the equinox that Mr. Van Flandern discussed earlier. Essentially, I just shifted the entire lunar ephemeris by 1^{s}44 in the independent argument, time, relative to the planetary ephemerides. When lunar spacecraft data were reduced using this ephemeris, the station longitudes shifted by some 28 to 30 m in the desired direction. Thus, it appears that the inconsistencies in the solutions for tracking station longitudes have been reduced by this means. This suggests, but does not yet prove, that the equinox or mean longitude correction given by Van Flandern is correct to at least one significant figure. We are planning, sometime this Fall, to construct an ephemeris that rigorously incorporates the entire set of Van Flandern's corrections, as well as modern values of the planetary masses. This experimental ephemeris will be used, among other purposes, to test the proposed corrections to the lunar theory by application to spacecraft tracking data.

It has recently been suggested that something is seriously wrong with the current ephemeris, well beyond anything discussed above. The MIT Lincoln Laboratory has made observations of the moon by doppler radar techniques (Ref. 31). These seem to indicate discrepancies, when compared with the lunar ephemeris, that are 10 or 20 times the size of those that seem to be indicated by JPL work with spacecraft data. We have only begun to examine this problem, and we do not yet know where the difficulty lies.

Later in this calendar year, after the differential correction problem is under control, we intend to start using the different observation types that are available to us, not only the conventional astronomical types, but also radar and (eventually) laser data.

Discussion

Lieske: You stated that it was less expensive to do several integrations than to integrate the variational equations. I would think that just the opposite is true.

Mulholland: I meant to indicate that, in terms of the easily modified programs *available to us at that time*, the finite difference approach was more economical. I did not mean to indicate it as a general principle.

Lowrey: In my satellite work, I have obtained two different solutions which give equally consistent results. In particular, I get two different values of the semi-major axis and the argument of perigee that give approximately the same residuals, so I think you can find several sets of corrections that will give equally good results.

Mulholland: That is not the case if you have an adequate distribution of observations. You must remember, too, that in the present work I am not dependent on the problem of observational distribution. Since the data being fitted are obtained from a theory, I can choose the "observation" times to be at whatever intervals I wish, over whatever period of time I wish.

Herrick: It seems to me that your results bear out my contention that you ought not to correct the mean motion. The moon has been observed for many centuries and the mean motion is very well known, and I think this is the quantity that is wanted here. I think that the fact that your residuals went down within the correction interval and then grew again is a probable indication of this.

Mulholland: It is certainly an indication that there was something wrong with the mean motion. However, the rest of your comment is not true, simply because we are not correcting the mean elements, but rather the osculating state vector at the epoch.

Herrick: But this embodies the mean motion.

Mulholland: Yes, but it also embodies all of the perturbations, and it is precisely these perturbations that are not well enough known.

N69-34643

Numerical Integration of the Lunar Orbit

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A least squares solution was made for the orbits of the nine major planets and the moon by fitting to optical observations spanning more than 50 years. Out of a total of 21,000 data points, almost 4,000 are lunar observations. All orbits were computed by numerical integration with a step size of 0ᵈ4. Originally, we had in mind to concentrate on the planets, intending to get the lunar orbit only accurately enough to properly describe the motion of the earth around the earth–moon barycenter. Then we began to realize that, after all, we were treating the orbital motion of the moon just as if it were another planet. We subsequently decided to improve the lunar orbit also to whatever degree of accuracy the observations would permit.

The moon proved considerably more difficult to handle than the planets, particularly in the differential correction part of the algorithm. Various forms of approximate partial derivatives had to be abandoned. We finally resorted to the relatively expensive numerically integrated partials.

The solution available today for the planets is believed to be very close to the optimum obtainable with optical data.

The rms in the residuals of $\alpha \cos \delta$ and δ for the entire solar system is 0″9. The lunar residuals are shown in Figs. 1 and 2. Figure 1 shows that a minor problem remains in the right ascension of the moon. There is a distinct signal with a period of approximately 18 years that has yet to be identified. Hence, the rms for the moon in $\alpha \cos \delta$ is still a rather large 1″66, and 1″07 in δ. The secular trend visible in Fig. 1 is of no concern; experience has shown that this bias will be removed by another iteration or two.

The tables of elements and constants (Tables 1 and 2) contain all the information needed to duplicate our orbits. The relatively large residuals for the moon are, of course, reflected in the standard deviations given for the lunar elements.

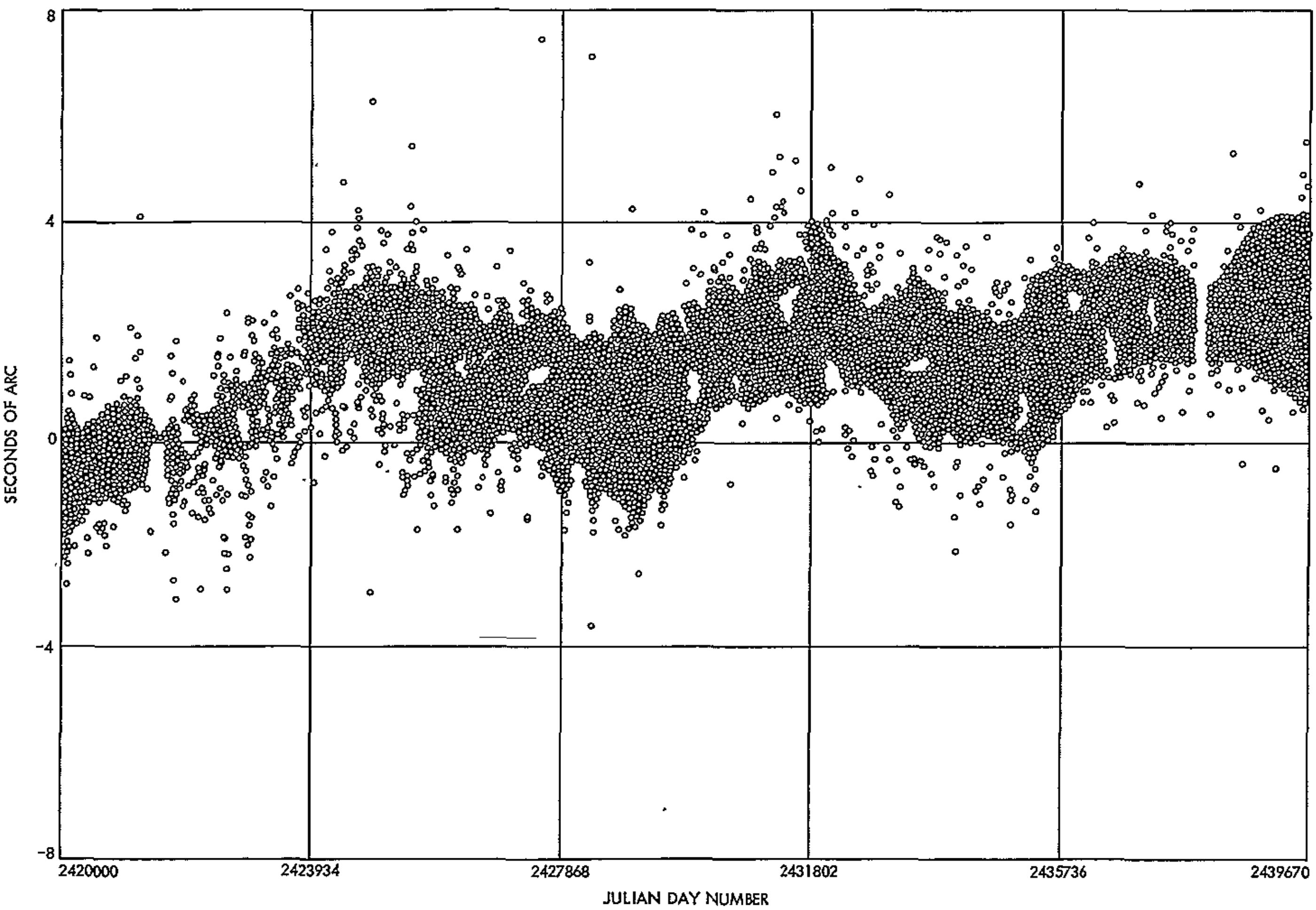

Fig. 1. Moon residuals in right ascension times cos delta, run 410, iteration 1

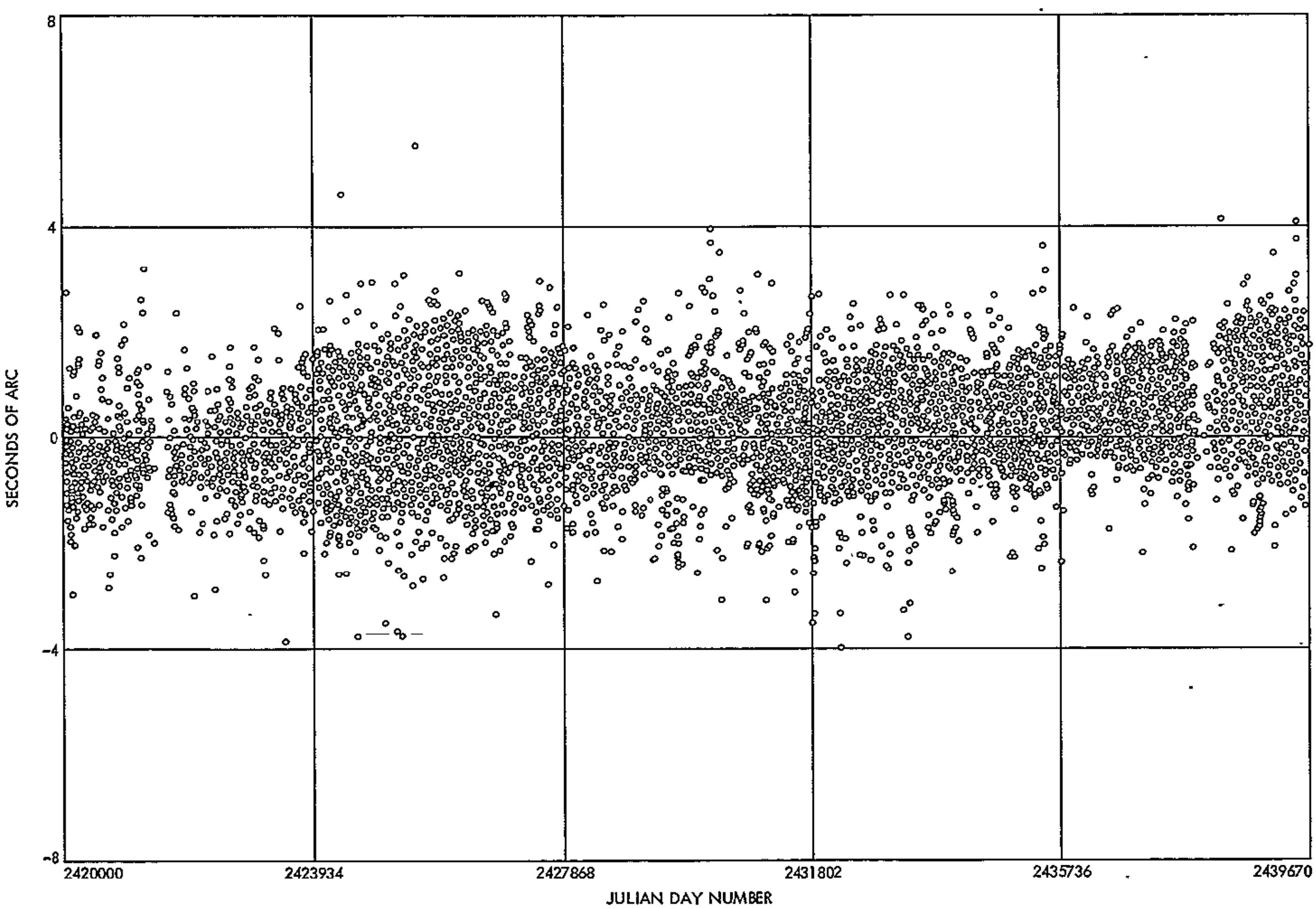

Fig. 2. Moon residuals in declination, run 410, iteration 1

Table 1. Reciprocal masses

Planet	Mass	Reference
Mercury	0.6021×10^7	Shapiro, 1967
Venus	0.40825×10^6	Shapiro, 1967
Earth	0.332945×10^6	Shapiro, 1967
Mars	0.31112×10^7	Shapiro, 1967
Jupiter	0.1047355×10^4	Astron. Papers, Vol. XII
Saturn	0.35016×10^4	Astron. Papers, Vol. XII
Uranus	0.22869×10^5	Astron. Papers, Vol. XII
Neptune	0.19314×10^5	Astron. Papers, Vol. XII
Pluto	0.36×10^6	Astron. Papers, Vol. XII
Moon	0.27069427×10^8	Shapiro, 1967

$k = 0.017\ 2020\ 9895$.
$c = 299\ 792.5$ km/s (used for light travel time only).
1 AU = 149 597 900 km (used for light travel time only).

Table 2. Osculating elements at JD 242 0000.5 (mean ecliptic and equinox 1950.0[d])

Planet	a, AU	e	i	l	g	h
Mercury	0.387 097 842 9 ±3	0.205 624 4 ±3	7°006 20 ±2	324°153 15 ±9	28°836 4 ±2	47°783 8 ±2
Venus	0.723 325 565 5 ±5	0.006 855 69 ±9	3.394 443 ±7	271.977 6 ±8	54.566 9 ±8	76.331 2 ±1
Earth	0.999 416 601 7 ±3	0.016 945 61 ±3	0.003 033 ±3	225.239 2 ±1	92.859 0 ±575[b]	11.090 7 ±575[b]
Mars	1.523 662 705 ±1	0.093 219 38 ±4	1.852 892 ±4	49.430 16 ±2	285.680 7 ±1	49.279 5 ±1
Jupiter	5.202 966 05 ±3	0.048 091 69 ±5	1.307 500 ±6	279.422 71 ±6	273.533 9 ±3	99.861 0 ±3
Saturn	9.523 631 7 ±2	0.053 680 24 ±6	2.489 646 ±7	340.873 66 ±6	339.048 8 ±2	113.346 6 ±1
Uranus	19.280 407 ±4	0.044 254 7 ±1	0.773 721 ±6	128.117 1 ±3	100.551 3 ±6	73.804 0 ±5
Neptune	29.985 94 ±5	0.008 226 ±1	1.775 954 ±7	89.554 7 ±51[c]	254.710 1 ±52[c]	131.229 2 ±2
Pluto	39.383 ±1	0.249 779 ±4	17.182 73 ±5	248.743 ±6	114.401 ±4	109.584 13 ±7
Moon[a]	0.002 563 725 396 ±9	0.058 227 40 ±6	28.680 225 ±6	198.587 89 ±6	177.269 14 ±6	359.000 21 ±1

[a]Lunar elements are geocentric equatorial.
[b]Sum $g + h$ well defined.
[c]Sum $l + g$ well defined.
[d]Formal standard deviations given in units of last decimal quoted.

ET-UT Time Corrections for the Period 1627–1860[1]

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Summary

The corrections to the moon's elements are based on the time period 1627–1860, so that, in conjunction with the modern corrections, the author gets very accurate determinations of the motions of the elements. The following is a typical example of the corrections that have been derived are those for the ascending node of the moon

Period	Corrections
1860–1840	$-2''.8 \pm 2''.4$
1840–1820	-10.6 ± 3.4
1820–1800	-6.7 ± 5.6
1800–1780	-12.5 ± 7.6

Before 1780, all of the corrections are smaller than their probable errors, so they are significant only in a statistical sense. These figures may give some indication of the size of the corrections and of the probable errors involved in the full solution.

The combination of these corrections with the modern values gives interesting results for some of the parameters. For example, the secular motion of the obliquity of the ecliptic requires a correction

$$\Delta\dot{\epsilon} = -0''.4 \pm 0''.2$$

confirming the results of Duncombe (Ref. 32) and others. The corrections indicated to the rates of the node and perigee are

$$\Delta\dot{\Omega} = -6''.3 \pm 2$$

$$\Delta\dot{\tilde{\omega}} = -0''.1 \pm 1$$

referred to the ideal coordinate system of the solution rather than the FK4 system. These quantities presently exhibit large discrepancies between theoretical and observed values, but the above results seem to go a long way

[1]Summarized by T. C. Van Flandern (USNO) in the author's absence.

toward accounting for those discrepancies. You may recall that Eckert (Ref. 33) discussed this problem and concluded that, if one takes the old values literally, it seems necessary to believe that most of the mass of the moon is concentrated near the surface and the center is of much lower density. With the above corrections, one can believe that the surface regions have approximately the same mean density as the center.

The application of all of the corrections in the solution to the observations reduced the residuals by a factor of approximately two.

Appendix A
Resolution on the Astrographic Catalogue

The following resolution[1] was adopted unanimously by the participants:

"The conference is cognizant of the potential importance of the contributions of optical determination of space probe positions, for the case of planetary spacecraft as well as the now-demonstrated case of lunar vehicles. In order to accomplish such determinations, a large number of positions of reference stars, down to at least the 12th magnitude, is required on a well-defined fundamental system.

[1]As it appeared in *Bull. Am. Astronom. Soc.*, *1*, p. 167, 1969.

These can be obtained in two steps:

(1) Initiation of steps to obtain plate constants which will reduce the measured coordinates to positions in the FK4 system for those plates of the Astrographic Catalogue for which this has not yet been done, and

(2) A reobservation of the positions of the Astrographic Catalogue stars making fullest possible use of modern techniques.

The conference recommends that the means be made available for carrying these recommendations into execution."

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